Appendix A – Outputs for Estimating MoE

This Appendix presents methods of finding the Test Comparison Value (*TCV*) in order to have given confidence that the production meets or exceeds the Design Value (*DV*) for MoE.

In estimating the MoE of a batch, random samples are taken as representative of the batch.

The MoE properties found from testing and analysis of the sampled timber are taken as estimates of the batch properties.

Where the test results are greater than or equal to Test Comparison Values calculated from tables or equations in this Appendix, then the producer has the assigned confidence that the production batch meets or exceeds the Design Value.

The steps in using the appendix are as follows:

1. Decide on a level of confidence that the MoE of the material produced meets the Design Value.
2. Select the section appropriate for the analysis method (eg non-parametric, log-normal or estimates from average MSG) to be used.
3. Find the value of Test Comparison Value (*TCV*):
   - For tables, use the selected Confidence Level, sample size and estimated CoV of the grade to look up value of ***M***, and find $TCV = \boldsymbol{M} \times DV$. Interpolation is possible within the table.
   - For equations, use the required Confidence Level to select a value of *A*, and this is used to find the Test Comparison Value for any value of CoV or sample size.
   - For graphs, use the selected Confidence Level, sample size and estimated CoV of the grade to look up value of ***M***, and find $TCV = \boldsymbol{M} \times DV$. (It is more accurate to use either tables or equations, but the graphs provide a visual representation of trends.
4. Use the Test Comparison Value in the monitoring system.

For MoE, outputs have been produced giving information on:

- the mean MoE (Section A.1)
- the 5%ile MoE (Section A.2).

Refer to examples in Section 6 for further guidance.

A.1 Mean MoE

A.1.1 Multipliers *(M)* for **non-parametric** mean MoE

$$TCV = \boldsymbol{M} * DV$$

Tables give values of ***M***

Non Parametric Mean MoE					
CL 95%	CoV				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.063	1.080	1.097	1.124	1.173
10	1.044	1.055	1.067	1.085	1.116
20	1.030	1.038	1.046	1.059	1.080
30	1.025	1.031	1.037	1.047	1.064
50	1.019	1.024	1.029	1.036	1.049
100	1.013	1.017	1.020	1.025	1.034
200	1.009	1.012	1.014	1.018	1.024

Non Parametric Mean MoE					
CL 80%	CoV				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.032	1.040	1.048	1.061	1.083
10	1.022	1.028	1.034	1.042	1.057
20	1.016	1.019	1.023	1.029	1.040
30	1.013	1.016	1.019	1.024	1.032
50	1.010	1.012	1.015	1.018	1.025
100	1.007	1.009	1.010	1.013	1.017
200	1.005	1.006	1.007	1.009	1.012

Non Parametric Mean MoE					
CL 90%	CoV				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.048	1.061	1.074	1.095	1.130
10	1.034	1.043	1.051	1.065	1.089
20	1.024	1.030	1.036	1.045	1.061
30	1.019	1.024	1.029	1.037	1.049
50	1.015	1.019	1.022	1.028	1.038
100	1.010	1.013	1.016	1.020	1.026
200	1.007	1.009	1.011	1.014	1.019

Non Parametric Mean MoE					
CL 75%	CoV				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.025	1.032	1.038	1.048	1.065
10	1.018	1.022	1.027	1.034	1.045
20	1.012	1.016	1.019	1.024	1.032
30	1.010	1.013	1.015	1.019	1.026
50	1.008	1.010	1.012	1.015	1.020
100	1.006	1.007	1.008	1.010	1.014
200	1.004	1.005	1.006	1.007	1.010

Non Parametric Mean MoE					
CL 85%	CoV				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.039	1.049	1.059	1.075	1.103
10	1.027	1.034	1.041	1.052	1.071
20	1.019	1.024	1.029	1.036	1.049
30	1.016	1.019	1.023	1.029	1.040
50	1.012	1.015	1.018	1.023	1.030
100	1.008	1.011	1.013	1.016	1.021
200	1.006	1.007	1.009	1.011	1.015

The estimation of mean MoE using non-parametric methods is very simple.

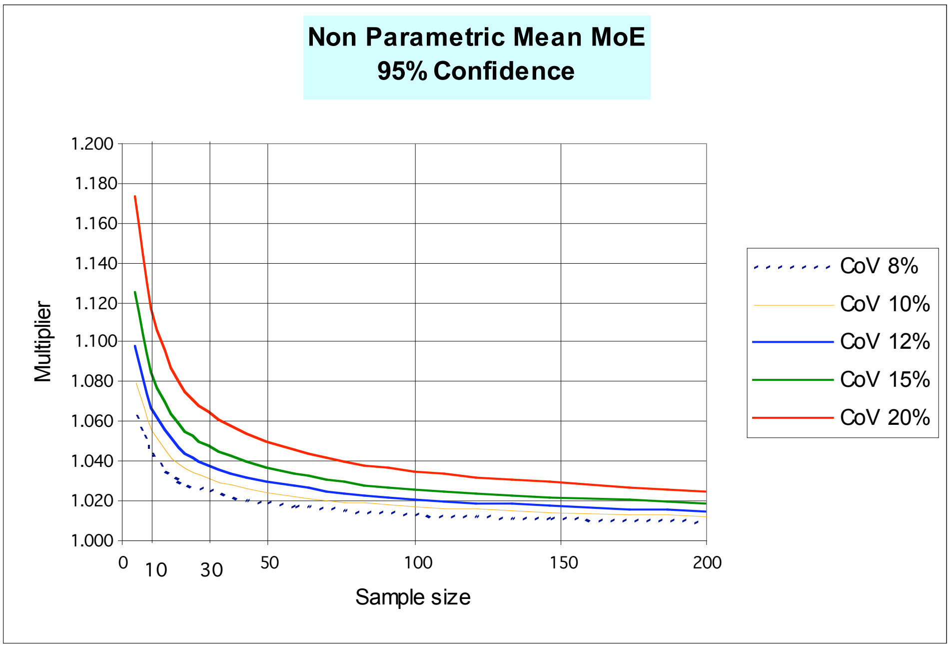
- The estimate is the average of all of the MoE data.
- No distribution is fitted.
- The AVERAGE function in msExcel can be used.

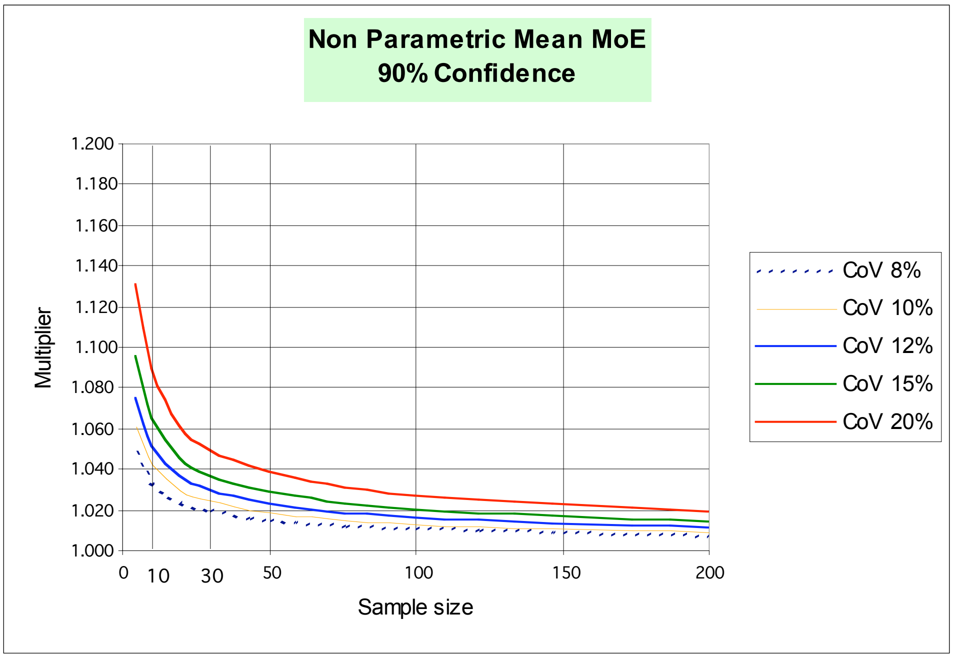
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

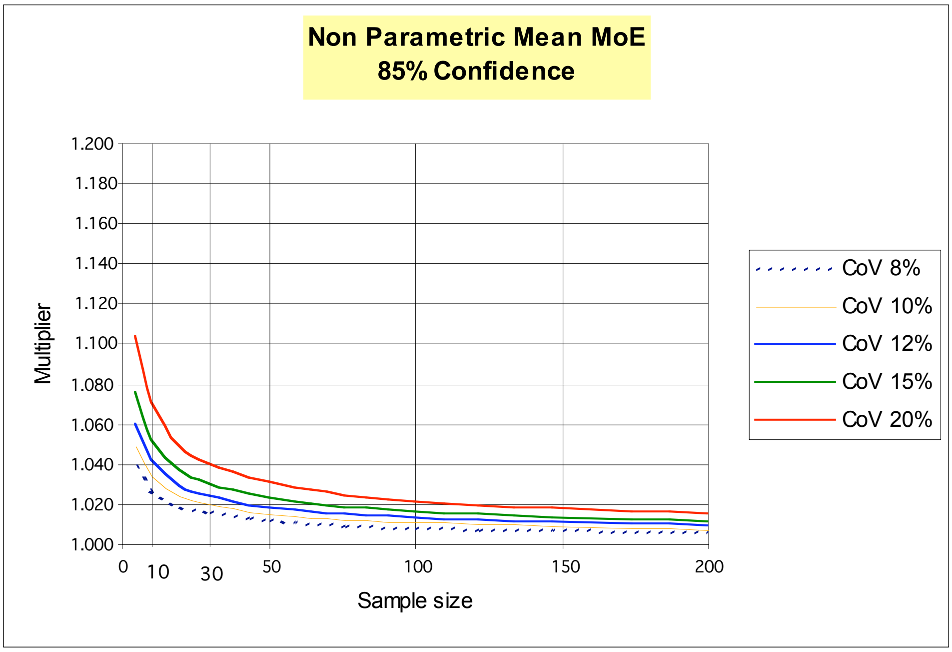
Equation – Non-parametric mean MoE

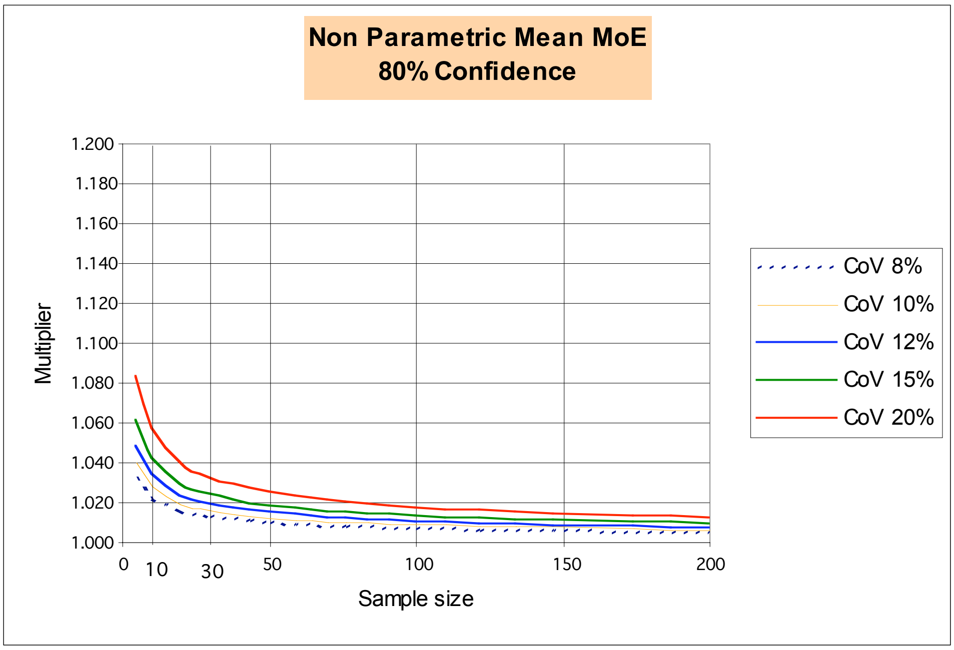
$$TCV = DV\left[\frac{1}{1 + A\frac{CoV}{\sqrt{n}}}\right]$$

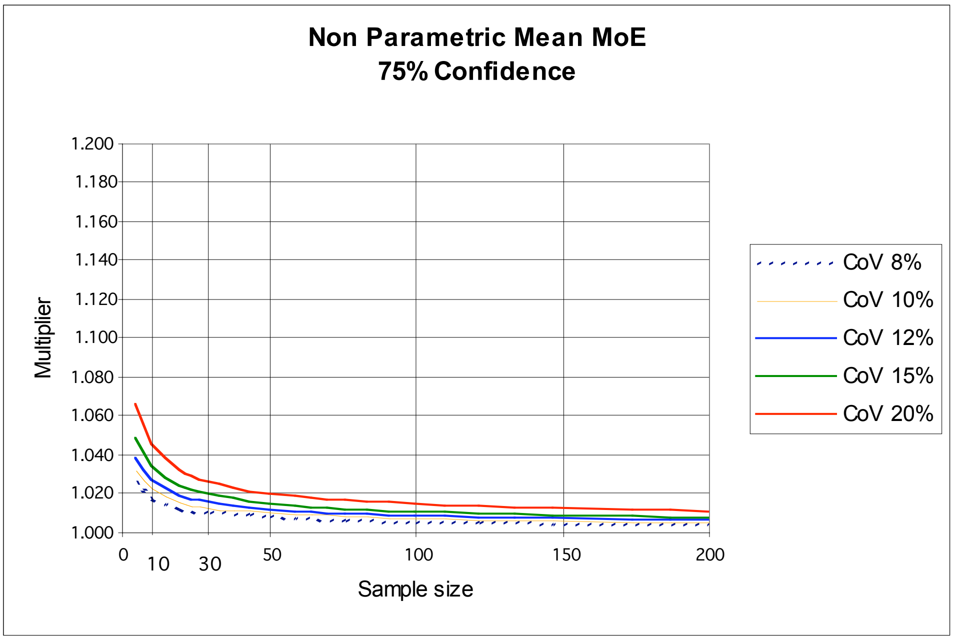
CL	*Value of A*
95%	-1.649
90%	-1.290
85%	-1.045
80%	-0.854
75%	-0.686


Non Parametric Mean MoE
95% Confidence
Multiplier
1.200
1.180
1.160
1.140
1.120
1.100
1.080
1.060
1.040
1.020
1.000
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Non Parametric Mean MoE
90% Confidence
Multiplier
1.200
1.180
1.160
1.140
1.120
1.100
1.080
1.060
1.040
1.020
1.000
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Non Parametric Mean MoE
85% Confidence
Multiplier
1.200
1.180
1.160
1.140
1.120
1.100
1.080
1.060
1.040
1.020
1.000
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Non Parametric Mean MoE
80% Confidence
1.200
1.180
1.160
1.140
1.120
1.100
1.080
1.060
1.040
1.020
1.000
Multiplier
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Non Parametric Mean MoE
75% Confidence
1.200
1.180
1.160
1.140
1.120
1.100
1.080
1.060
1.040
1.020
1.000
Multiplier
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%

A.1.2 Multipliers *(M)* for **Log-normal** mean MoE

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

Log-Normal Mean MoE					
CL 95%	**CoV**				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.063	1.080	1.098	1.125	1.174
10	1.044	1.055	1.067	1.085	1.117
20	1.031	1.038	1.047	1.059	1.080
30	1.025	1.031	1.038	1.048	1.064
50	1.019	1.024	1.029	1.036	1.049
100	1.013	1.017	1.020	1.025	1.034
200	1.009	1.012	1.014	1.018	1.024

Log-Normal Mean MoE					
CL 80%	**CoV**				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.032	1.040	1.048	1.061	1.083
10	1.022	1.028	1.034	1.043	1.058
20	1.016	1.020	1.024	1.030	1.040
30	1.013	1.016	1.019	1.024	1.032
50	1.010	1.012	1.015	1.019	1.025
100	1.007	1.009	1.010	1.013	1.018
200	1.005	1.006	1.007	1.009	1.012

Log-Normal Mean MoE					
CL 90%	**CoV**				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.049	1.062	1.075	1.095	1.131
10	1.034	1.043	1.052	1.066	1.089
20	1.024	1.030	1.036	1.045	1.062
30	1.019	1.024	1.029	1.037	1.050
50	1.015	1.019	1.023	1.028	1.038
100	1.010	1.013	1.016	1.020	1.027
200	1.007	1.009	1.011	1.014	1.019

Log-Normal Mean MoE					
CL 75%	**CoV**				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.025	1.032	1.039	1.049	1.066
10	1.018	1.022	1.027	1.034	1.046
20	1.013	1.016	1.019	1.024	1.032
30	1.010	1.013	1.015	1.019	1.026
50	1.008	1.010	1.012	1.015	1.020
100	1.006	1.007	1.008	1.011	1.014
200	1.004	1.005	1.006	1.007	1.010

Log-Normal Mean MoE					
CL 85%	**CoV**				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.039	1.049	1.060	1.076	1.104
10	1.027	1.034	1.042	1.053	1.071
20	1.019	1.024	1.029	1.037	1.049
30	1.016	1.020	1.024	1.030	1.040
50	1.012	1.015	1.018	1.023	1.031
100	1.008	1.011	1.013	1.016	1.021
200	1.006	1.007	1.009	1.011	1.015

The estimation of mean MoE using a log-normal distribution through the data makes use of the natural logarithms of the raw MoE data (ln(MoE)). An extra column needs to be introduced to an analysis spreadsheet. This same column and many of the calculations necessary for this method can be used to estimate the 5%ile MoE from the log-normal distribution.

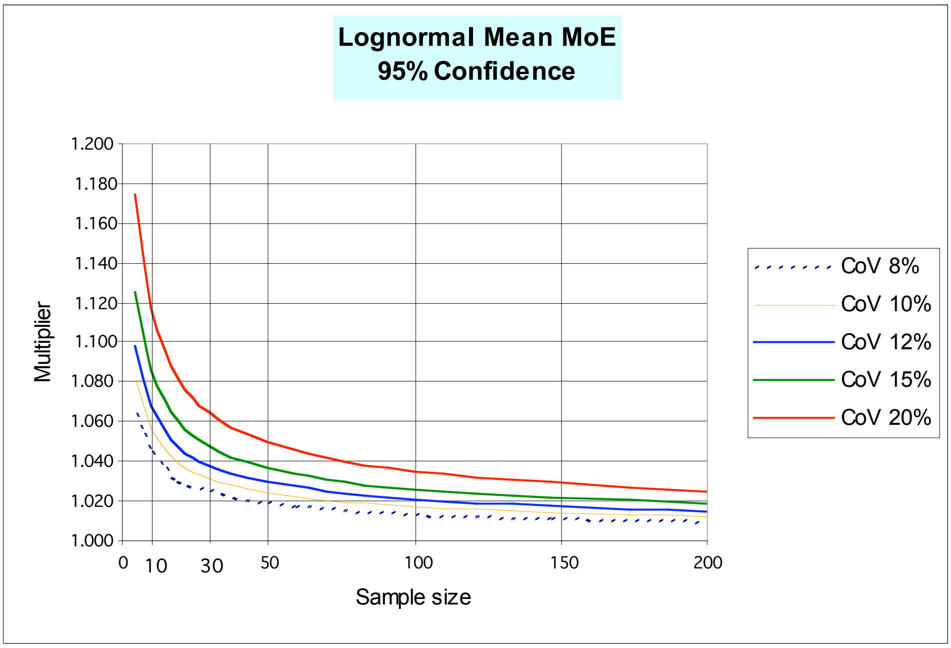
- The estimate of the mean MoE is based on the average and the standard deviation of the ln(MoE)data.
- The mean MoE can be found using (*eqn C.9*).
- The LN, EXP, AVERAGE and STDEV functions in msExcel are used.

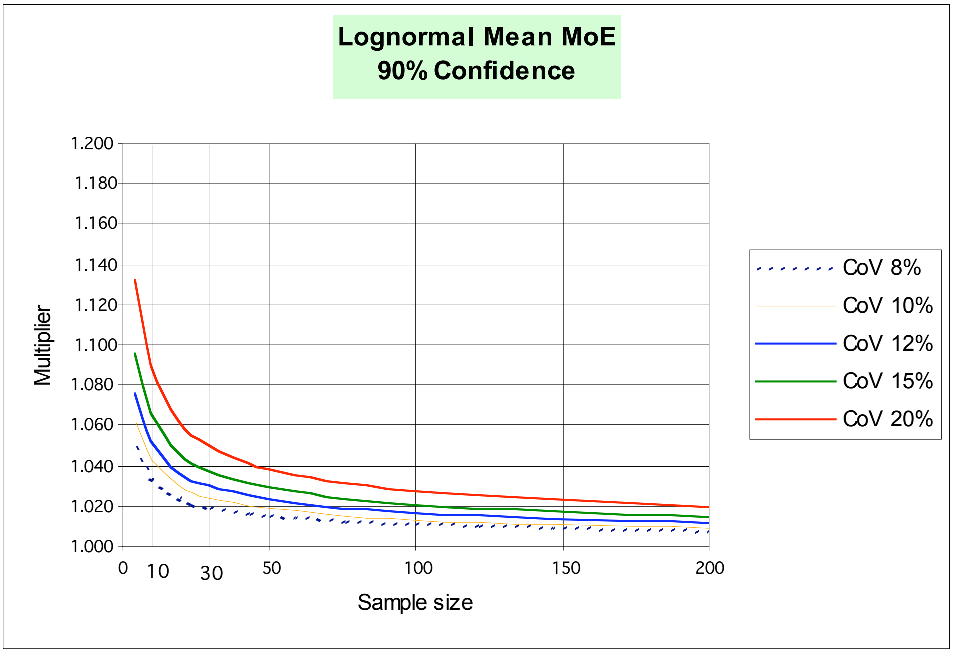
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

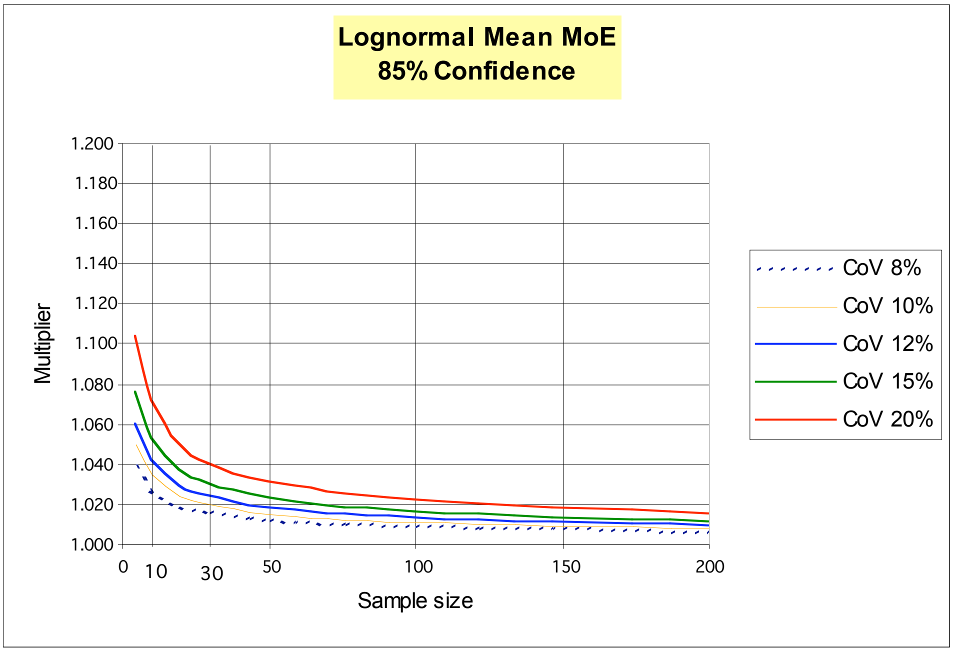
Equation – Log-normal mean MoE

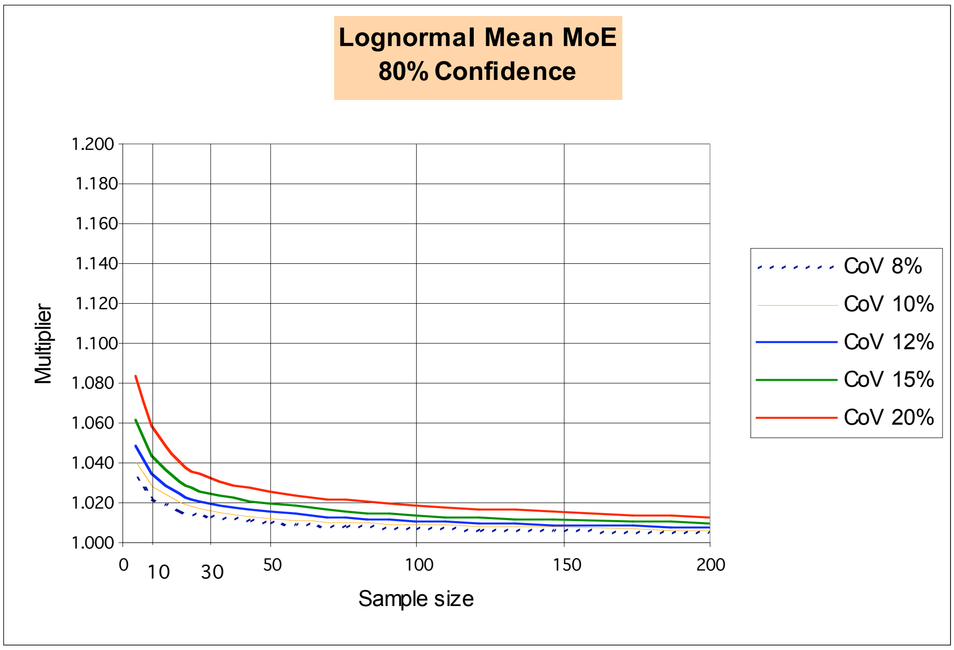
$$TCV = DV\left[\frac{1}{1 + A\frac{CoV}{\sqrt{n}}}\right]$$

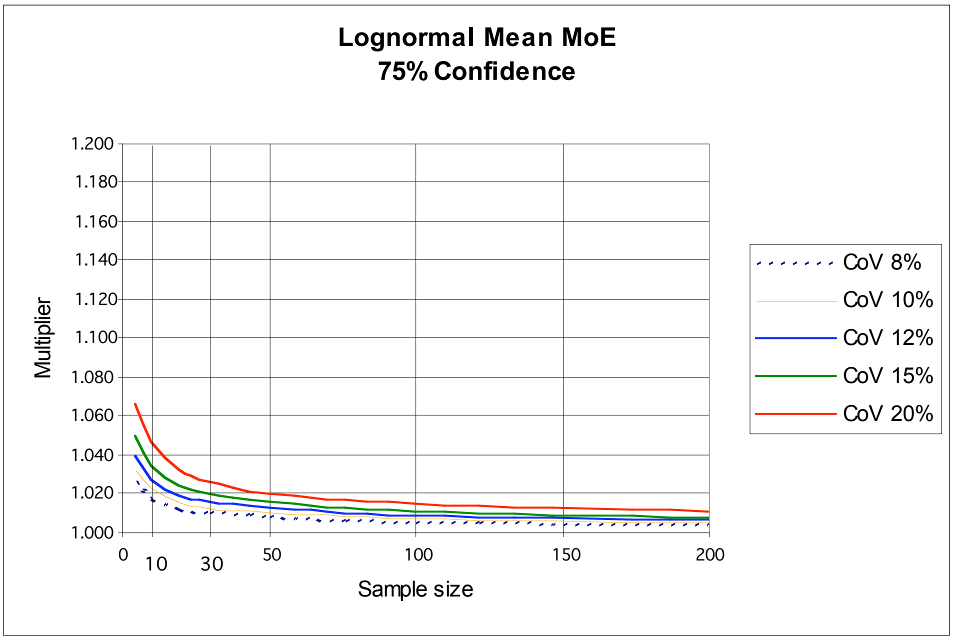
CL	*Value of A*
95%	-1.657
90%	-1.297
85%	-1.052
80%	-0.861
75%	-0.693


Lognormal Mean MoE
95% Confidence
Multiplier
1.200
1.180
1.160
1.140
1.120
1.100
1.080
1.060
1.040
1.020
1.000
0 10 30 50 100 150 200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Lognormal Mean MoE
90% Confidence
Multiplier
1.200
1.180
1.160
1.140
1.120
1.100
1.080
1.060
1.040
1.020
1.000
0 10 30 50 100 150 200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Lognormal Mean MoE
85% Confidence
Multiplier
1.200
1.180
1.160
1.140
1.120
1.100
1.080
1.060
1.040
1.020
1.000
0 10 30 50 100 150 200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Lognormal Mean MoE
80% Confidence
1.200
1.180
1.160
1.140
1.120
1.100
1.080
1.060
1.040
1.020
1.000
Multiplier
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Lognormal Mean MoE
75% Confidence
1.200
1.180
1.160
1.140
1.120
1.100
1.080
1.060
1.040
1.020
1.000
Multiplier
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%

A.1.3 Multipliers *(M)* for **MSG estimate** of Mean MoE

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

MSG Estimate of Mean MoE					
CL 95%	minMSG/avgMSG				
Sample size	**65%**	**70%**	**75%**	**80%**	**85%**
50	1.088	1.072	1.057	1.042	1.028
100	1.075	1.062	1.048	1.034	1.021
200	1.067	1.054	1.041	1.029	1.017
500	1.060	1.048	1.036	1.024	1.013
1000	1.056	1.044	1.033	1.022	1.011
2000	1.053	1.042	1.031	1.020	1.009
5000	1.051	1.040	1.029	1.018	1.008
10000	1.050	1.039	1.028	1.018	1.007

MSG Estimate of Mean MoE					
CL 80%	minMSG/avgMSG				
Sample size	**65%**	**70%**	**75%**	**80%**	**85%**
50	1.068	1.054	1.042	1.029	1.017
100	1.061	1.049	1.037	1.025	1.014
200	1.057	1.045	1.034	1.022	1.011
500	1.054	1.042	1.031	1.020	1.009
1000	1.052	1.040	1.029	1.019	1.008
2000	1.050	1.039	1.028	1.018	1.007
5000	1.049	1.038	1.028	1.017	1.007
10000	1.049	1.038	1.027	1.017	1.006

MSG Estimate of Mean MoE					
CL90%	minMSG/avgMSG				
Sample size	**65%**	**70%**	**75%**	**80%**	**85%**
50	1.078	1.064	1.050	1.036	1.023
100	1.069	1.056	1.043	1.030	1.018
200	1.063	1.050	1.038	1.026	1.014
500	1.057	1.045	1.034	1.022	1.011
1000	1.054	1.043	1.031	1.020	1.010
2000	1.052	1.041	1.030	1.019	1.008
5000	1.050	1.039	1.028	1.018	1.007
10000	1.049	1.038	1.028	1.017	1.007

MSG Estimate of Mean MoE					
CL75%	minMSG/avgMSG				
Sample size	**65%**	**70%**	**75%**	**80%**	**85%**
50	1.063	1.051	1.039	1.027	1.015
100	1.059	1.047	1.035	1.023	1.012
200	1.055	1.044	1.032	1.021	1.010
500	1.052	1.041	1.030	1.019	1.009
1000	1.051	1.040	1.029	1.018	1.008
2000	1.050	1.039	1.028	1.017	1.007
5000	1.049	1.038	1.027	1.017	1.007
10000	1.048	1.037	1.027	1.017	1.006

MSG Estimate of Mean MoE					
CL85%	minMSG/avgMSG				
Sample size	**65%**	**70%**	**75%**	**80%**	**85%**
50	1.072	1.059	1.045	1.032	1.020
100	1.065	1.052	1.040	1.027	1.016
200	1.060	1.047	1.036	1.024	1.013
500	1.055	1.043	1.032	1.021	1.010
1000	1.053	1.041	1.030	1.019	1.009
2000	1.051	1.040	1.029	1.018	1.008
5000	1.050	1.039	1.028	1.017	1.007
10000	1.049	1.038	1.027	1.017	1.007

The estimation of mean MoE using the average MSG of each piece in the period of production relies on collection of data already available in most Machine Stress Graders.

- The average MSG of each piece is averaged in the control unit of the MSG and is usually displayed continuously, and is recorded at the end of each run.
- The value recorded is simply compared with the Test Comparison Value.

The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

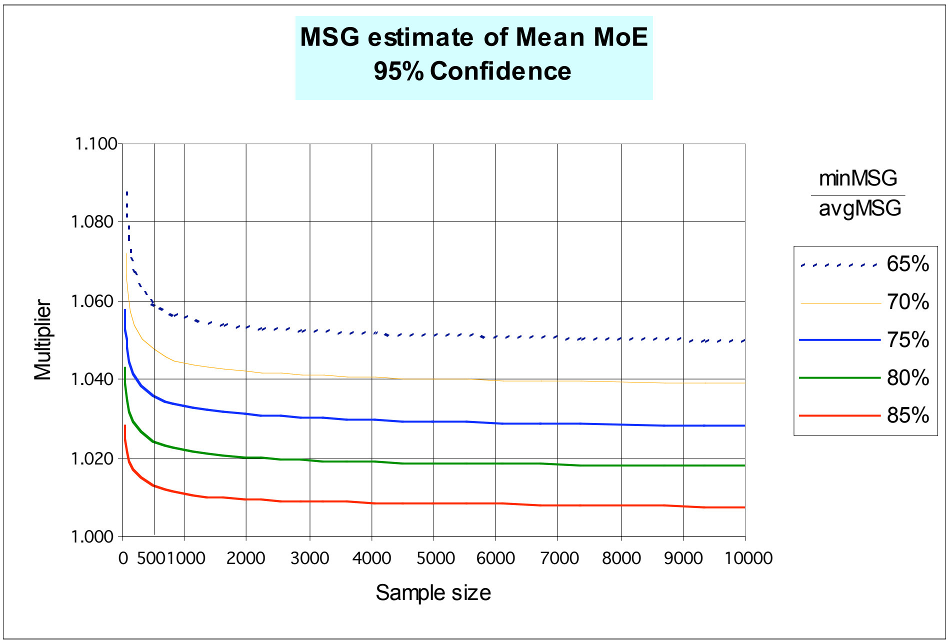
Equation – MSG estimate of mean MoE

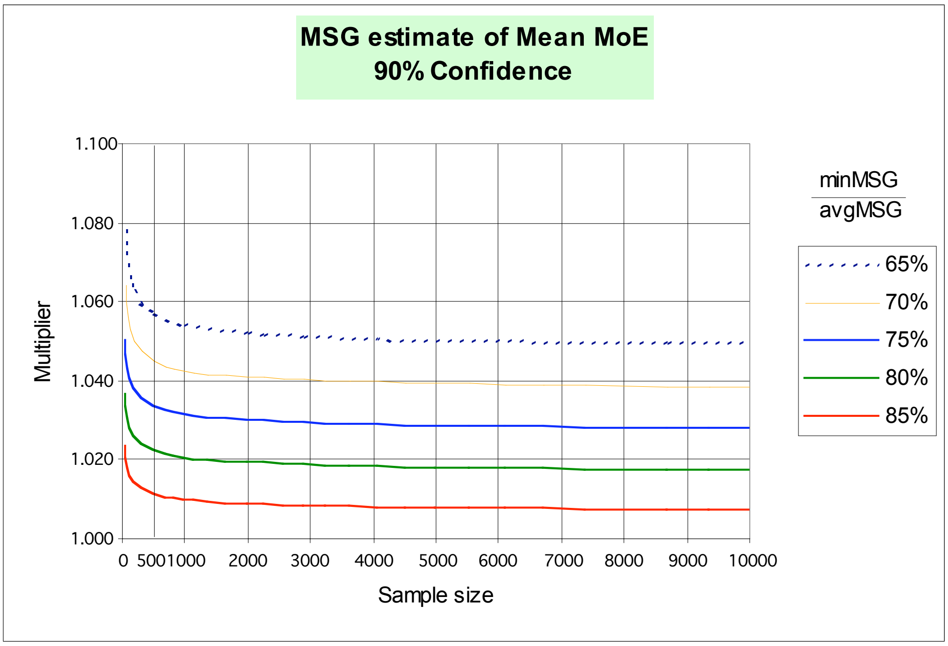
$$TCV = DV\left[\frac{1}{B\left(1 + A\frac{CoV}{\sqrt{n}}\right)}\right]$$

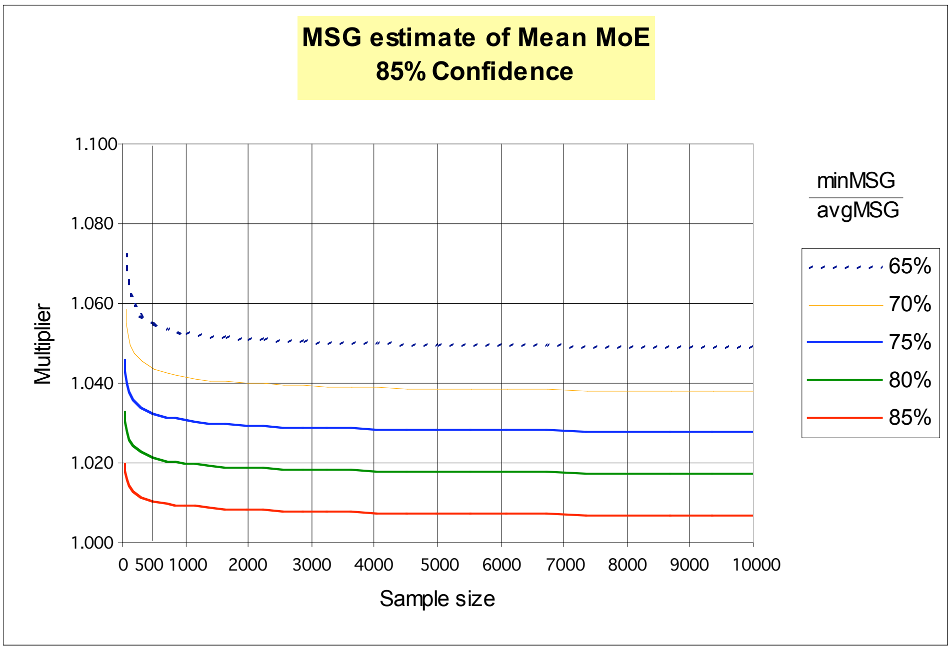
$$B = 0.827 + 0.197\left(\frac{\min MSG}{avg\ MSG}\right)$$

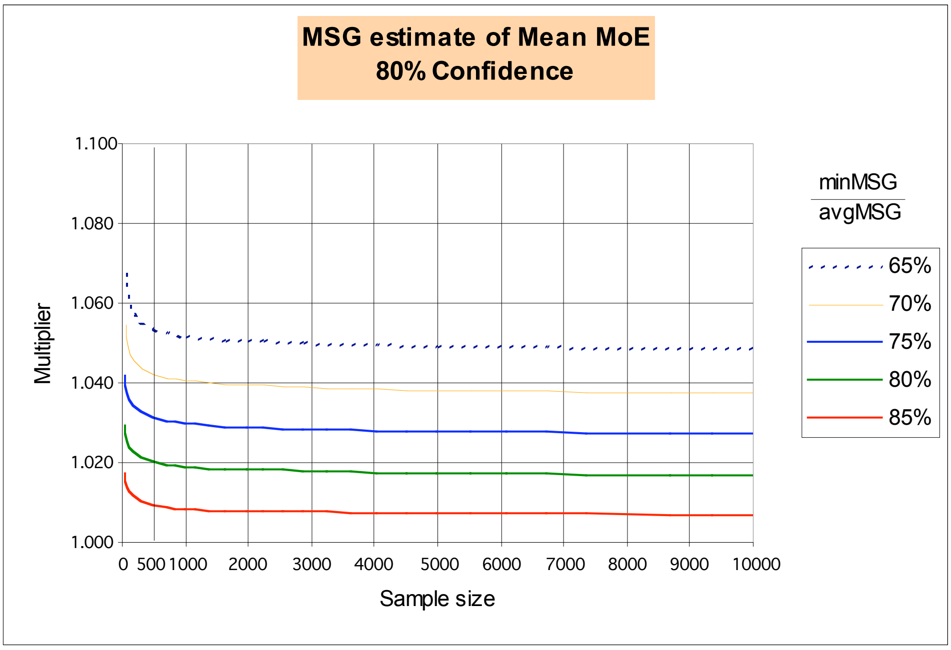
$$CoV = 0.377 - 0.334\left(\frac{\min MSG}{avg\ MSG}\right)$$

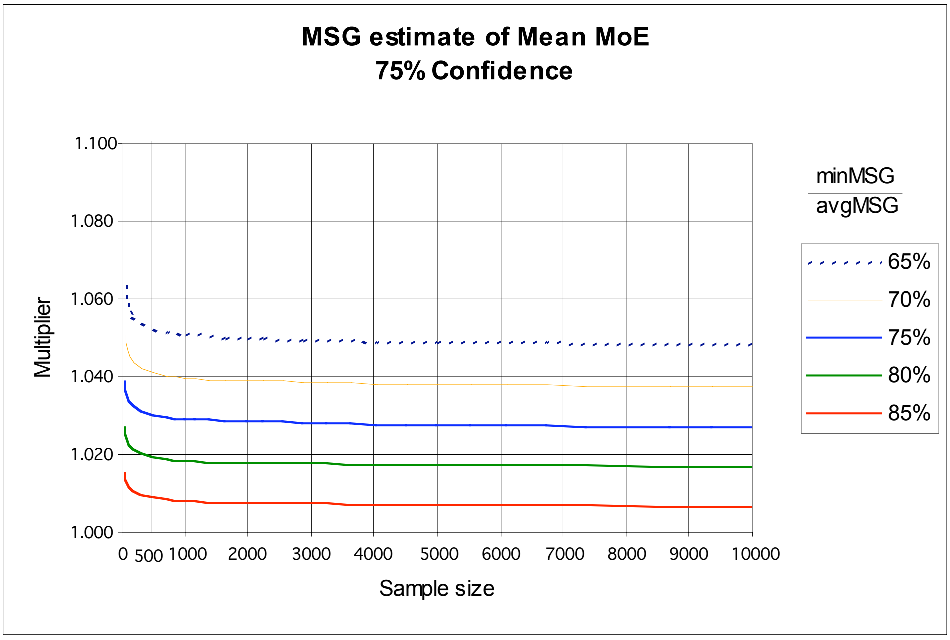
CL	*Value of A*
95%	-1.645
90%	-1.282
85%	-1.036
80%	-0.842
75%	-0.674


MSG estimate of Mean MoE
95% Confidence
1.100
1.080
1.060
1.040
1.020
1.000
Multiplier
0 5001000 2000 3000 4000 5000 6000 7000 8000 9000 10000
Sample size
minMSG
avgMSG
65%
70%
75%
80%
85%


MSG estimate of Mean MoE
90% Confidence
1.100
1.080
1.060
1.040
1.020
1.000
Multiplier
0 5001000 2000 3000 4000 5000 6000 7000 8000 9000 10000
Sample size
minMSG
avgMSG
65%
70%
75%
80%
85%


MSG estimate of Mean MoE
85% Confidence
1.100
1.080
1.060
1.040
1.020
1.000
Multiplier
0 500 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000
Sample size
minMSG
avgMSG
65%
70%
75%
80%
85%


MSG estimate of Mean MoE
80% Confidence
Multiplier
1.100
1.080
1.060
1.040
1.020
1.000
0
500
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Sample size
minMSG
avgMSG
65%
70%
75%
80%
85%


MSG estimate of Mean MoE
75% Confidence
Multiplier
1.100
1.080
1.060
1.040
1.020
1.000
0
500
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Sample size
minMSG
avgMSG
65%
70%
75%
80%
85%

A.2 5%ile MoE

A.2.1 Multipliers *(M)* for **Non-parametric** 5%ile MoE

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

Non Parametric 5%ile MoE					
CL 95%	CoV				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.155	1.201	1.252	1.336	1.504
10	1.105	1.134	1.166	1.216	1.311
20	1.072	1.091	1.112	1.144	1.201
30	1.058	1.073	1.089	1.114	1.159
50	1.044	1.056	1.068	1.086	1.119
100	1.031	1.039	1.047	1.060	1.081
200	1.022	1.027	1.033	1.041	1.056

Non Parametric 5%ile MoE					
CL 80%	CoV				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.091	1.116	1.143	1.185	1.263
10	1.062	1.079	1.097	1.124	1.172
20	1.043	1.055	1.067	1.085	1.116
30	1.035	1.044	1.054	1.068	1.093
50	1.027	1.034	1.041	1.052	1.070
100	1.019	1.024	1.029	1.036	1.049
200	1.013	1.017	1.020	1.025	1.034

Non Parametric 5%ile MoE					
CL 90%	CoV				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.125	1.162	1.200	1.264	1.385
10	1.085	1.109	1.134	1.173	1.245
20	1.059	1.075	1.091	1.116	1.162
30	1.048	1.060	1.073	1.093	1.128
50	1.036	1.046	1.056	1.071	1.096
100	1.026	1.032	1.039	1.049	1.066
200	1.018	1.022	1.027	1.034	1.046

Non Parametric 5%ile MoE					
CL 75%	CoV				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.078	1.099	1.122	1.157	1.221
10	1.054	1.068	1.083	1.106	1.147
20	1.038	1.047	1.057	1.073	1.099
30	1.030	1.038	1.046	1.059	1.080
50	1.023	1.029	1.036	1.045	1.061
100	1.016	1.021	1.025	1.031	1.042
200	1.012	1.015	1.017	1.022	1.029

Non Parametric 5%ile MoE					
CL 85%	CoV				
Sample size	**8%**	**10%**	**12%**	**15%**	**20%**
5	1.106	1.136	1.167	1.218	1.314
10	1.073	1.092	1.113	1.145	1.203
20	1.050	1.064	1.077	1.098	1.136
30	1.041	1.051	1.062	1.079	1.108
50	1.031	1.039	1.048	1.060	1.082
100	1.022	1.027	1.033	1.042	1.056
200	1.015	1.019	1.023	1.029	1.039

The non-parametric 5%ile can be estimated for test data by ranking the test data, plotting it against its probability and interpolating to find the 5%ile value.

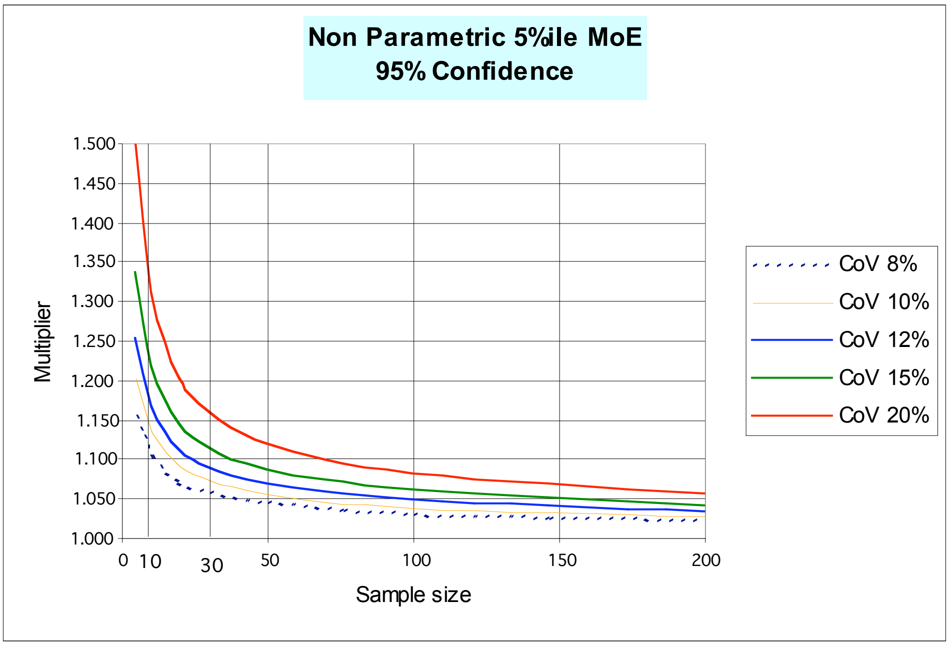
- Detail on the ranking and determination of the 5%ile value is given in Appendix C.2.1.
- The PERCENTILE function in msExcel can be used do this, but has some small errors that may lead to slightly different answers from more rigorously correct analysis methods.

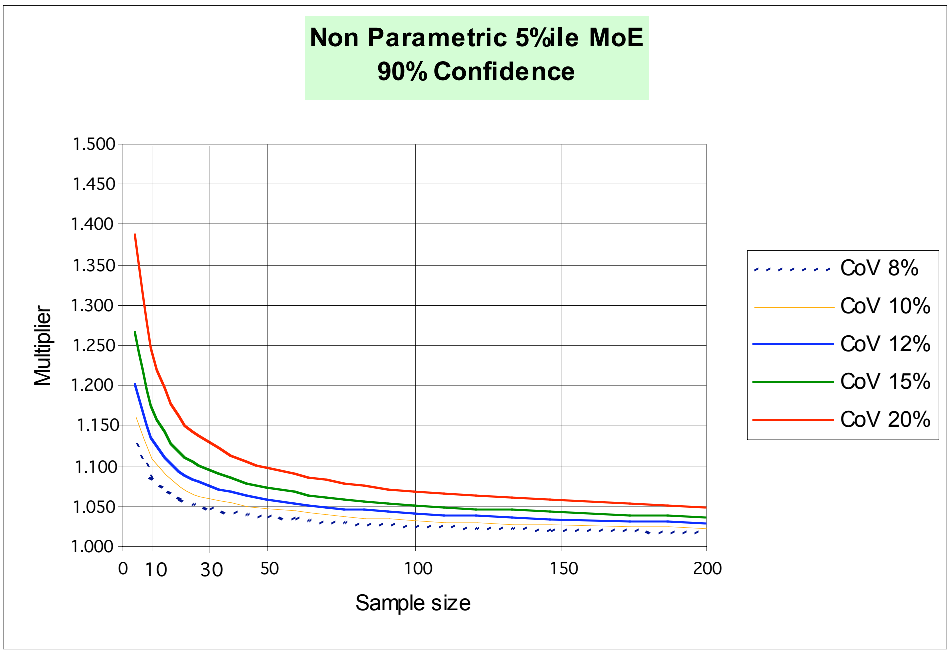
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

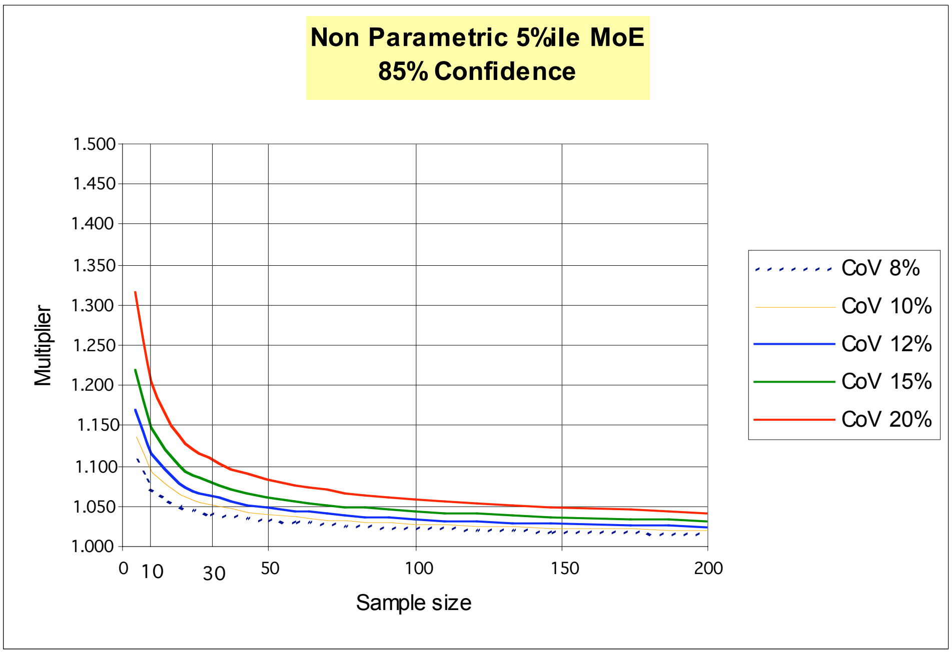
Equation – Non-parametric 5%ile MoE

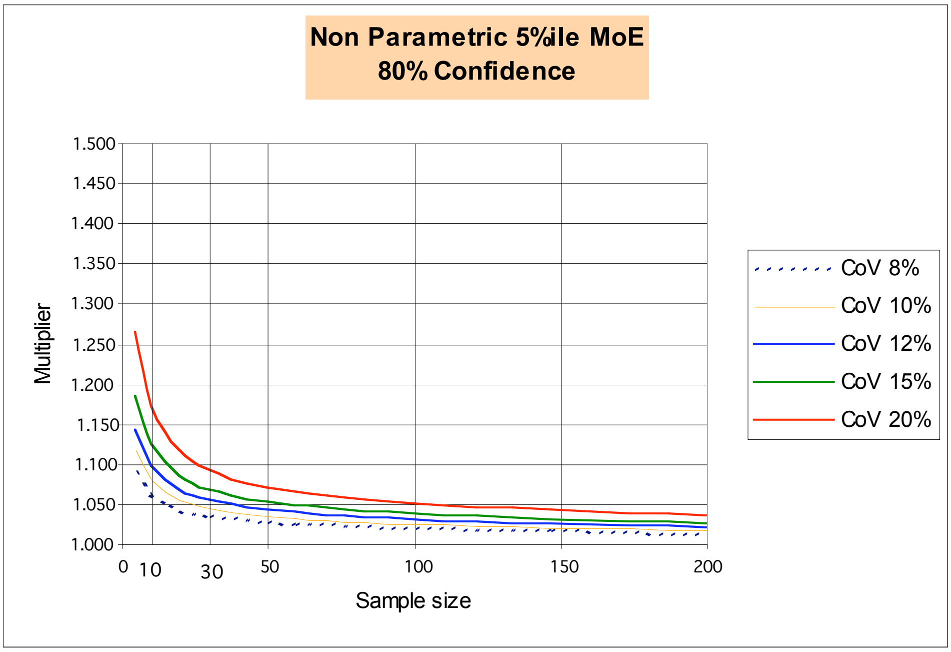
$$TCV = DV\left[\frac{1}{1 + A\frac{CoV}{\sqrt{n}}}\right]$$

CL	*Value of A*
95%	-3.747
90%	-3.110
85%	-2.672
80%	-2.325
75%	-2.024

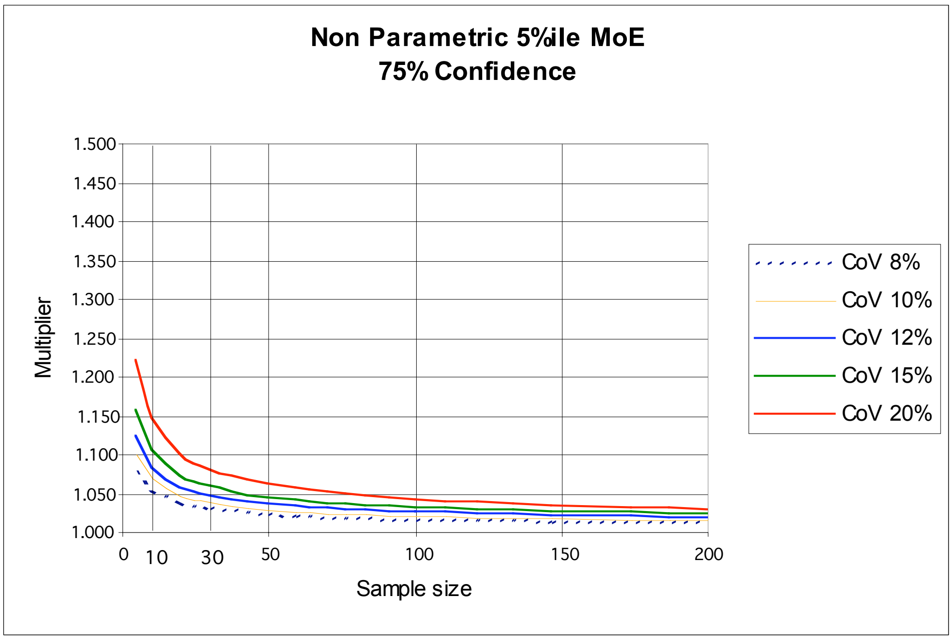

Non Parametric 5%ile MoE
95% Confidence
Multiplier
1.500
1.450
1.400
1.350
1.300
1.250
1.200
1.150
1.100
1.050
1.000
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Non Parametric 5%ile MoE
90% Confidence
Multiplier
1.500
1.450
1.400
1.350
1.300
1.250
1.200
1.150
1.100
1.050
1.000
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Non Parametric 5%ile MoE
85% Confidence
Multiplier
1.500
1.450
1.400
1.350
1.300
1.250
1.200
1.150
1.100
1.050
1.000
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%



Non Parametric 5%ile MoE
80% Confidence
Multiplier
1.500
1.450
1.400
1.350
1.300
1.250
1.200
1.150
1.100
1.050
1.000
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%



Non Parametric 5%ile MoE
75% Confidence
Multiplier
1.500
1.450
1.400
1.350
1.300
1.250
1.200
1.150
1.100
1.050
1.000
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%

A.2.2 Multipliers *(M)* for **Log-normal** 5%ile MoE

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

Log-normal 5%ile MoE					
CL 95%	CoV				
Sample size	8%	10%	12%	15%	20%
5	1.106	1.136	1.168	1.219	1.316
10	1.073	1.093	1.113	1.146	1.204
20	1.050	1.064	1.078	1.099	1.136
30	1.041	1.051	1.062	1.079	1.109
50	1.031	1.039	1.048	1.060	1.082
100	1.022	1.028	1.033	1.042	1.057
200	1.015	1.019	1.023	1.029	1.039

Log-normal 5%ile MoE					
CL 80%	CoV				
Sample size	8%	10%	12%	15%	20%
5	1.054	1.069	1.084	1.107	1.148
10	1.038	1.048	1.058	1.073	1.100
20	1.026	1.033	1.040	1.051	1.069
30	1.021	1.027	1.033	1.041	1.055
50	1.017	1.021	1.025	1.031	1.042
100	1.012	1.015	1.018	1.022	1.030
200	1.008	1.010	1.012	1.015	1.021

Log-normal 5%ile MoE					
CL 90%	CoV				
Sample size	8%	10%	12%	15%	20%
5	1.082	1.105	1.128	1.166	1.234
10	1.057	1.072	1.088	1.112	1.155
20	1.039	1.050	1.060	1.077	1.105
30	1.032	1.040	1.049	1.062	1.084
50	1.025	1.031	1.037	1.047	1.064
100	1.017	1.022	1.026	1.033	1.044
200	1.012	1.015	1.018	1.023	1.031

Log-normal 5%ile MoE					
CL 75%	CoV				
Sample size	8%	10%	12%	15%	20%
5	1.044	1.056	1.067	1.086	1.118
10	1.031	1.039	1.047	1.059	1.081
20	1.022	1.027	1.033	1.041	1.056
30	1.018	1.022	1.026	1.033	1.045
50	1.014	1.017	1.020	1.026	1.034
100	1.010	1.012	1.014	1.018	1.024
200	1.007	1.008	1.010	1.013	1.017

Log-normal 5%ile MoE					
CL 85%	CoV				
Sample size	8%	10%	12%	15%	20%
5	1.066	1.084	1.103	1.132	1.184
10	1.046	1.058	1.071	1.090	1.123
20	1.032	1.040	1.049	1.062	1.084
30	1.026	1.033	1.040	1.050	1.068
50	1.020	1.025	1.030	1.038	1.052
100	1.014	1.018	1.021	1.027	1.036
200	1.010	1.012	1.015	1.019	1.025

The estimation of 5%ile MoE using a log-normal distribution through the data makes use of the natural logarithms of the raw MoE data (ln(MoE)). An extra column needs to be introduced to an analysis spreadsheet. This same column and many of the calculations necessary for this method can also be used to estimate the mean MoE from the log-normal distribution.

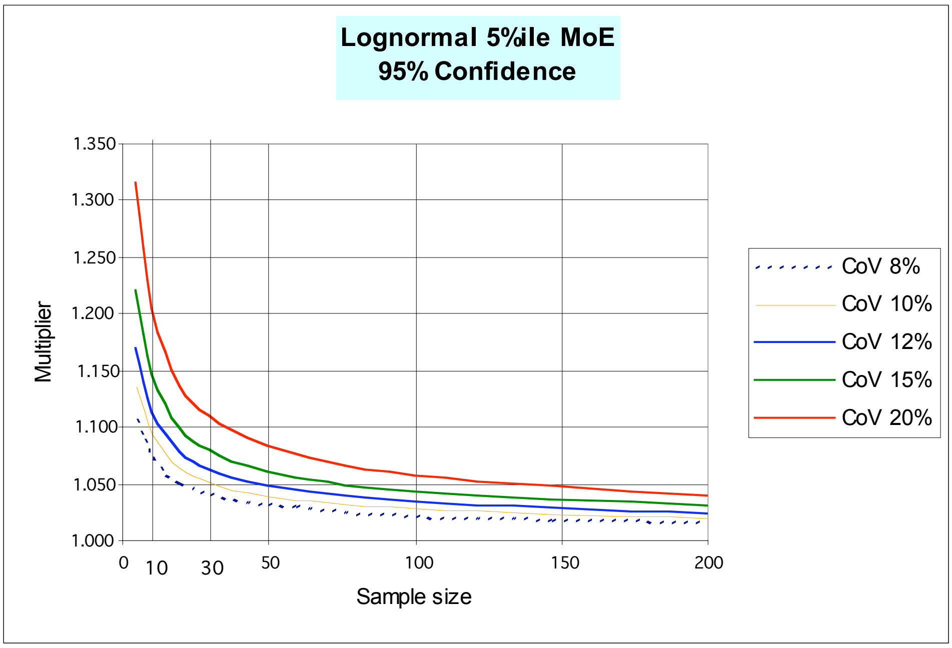
- The estimate of the 5%ile MoE is based on the average and the standard deviation of the ln(MoE)data.
- The 5%ile MoE can be found using (*eqn C.11*).
- The LN, EXP, AVERAGE and STDEV functions in msExcel are used.

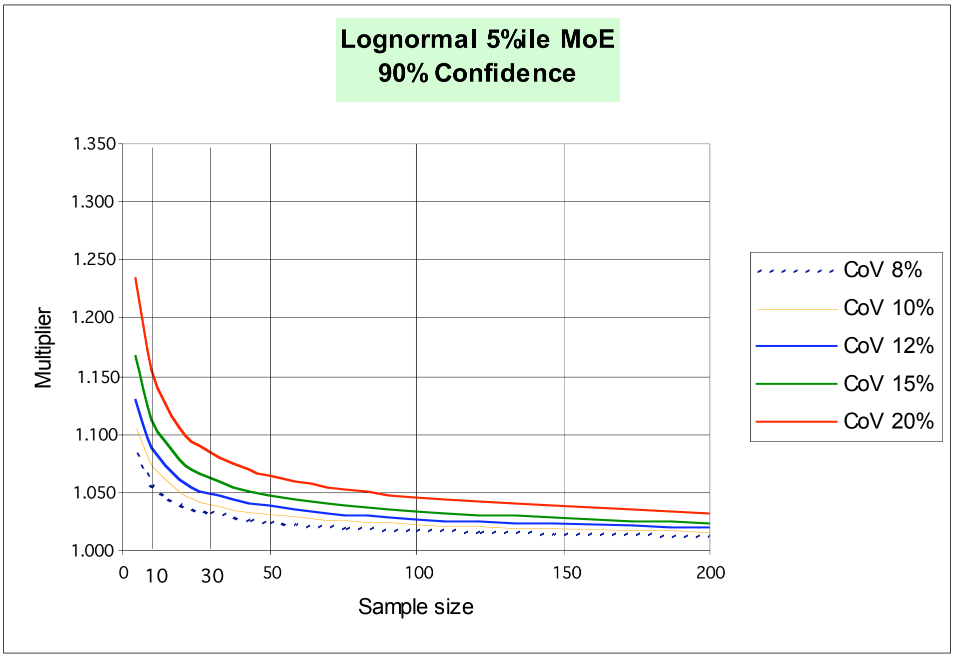
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

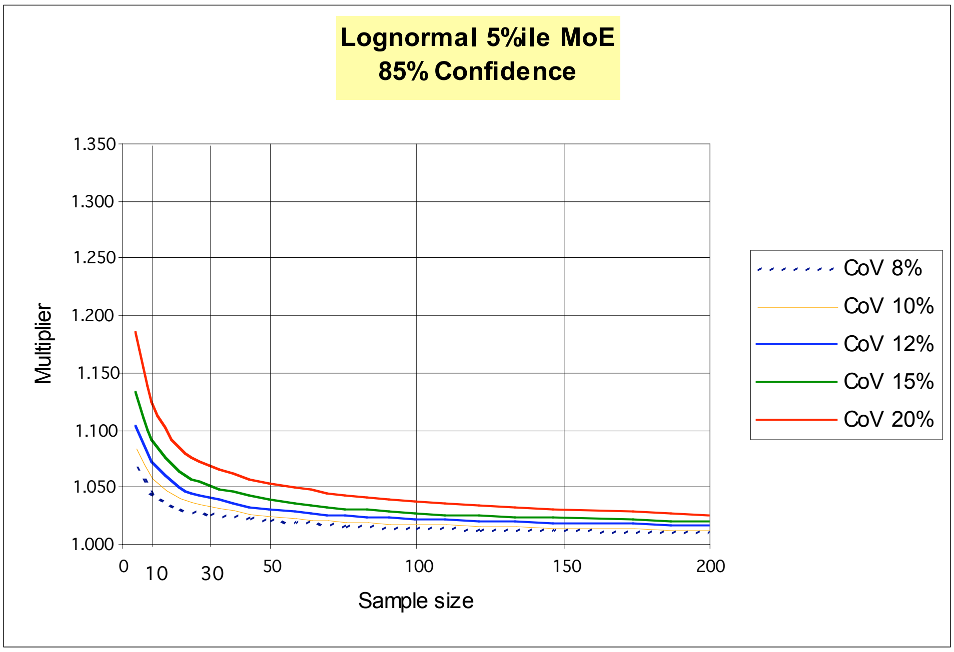
Equation – Log-normal 5%ile MoE

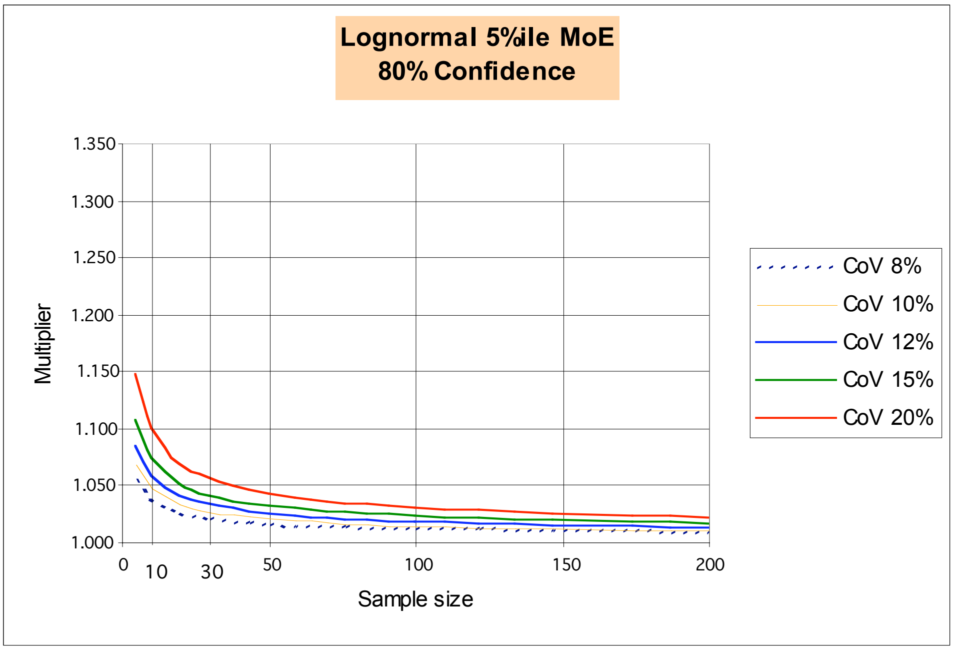
$$TCV = DV\left[\frac{1}{1 + A\frac{CoV}{\sqrt{n}}}\right]$$

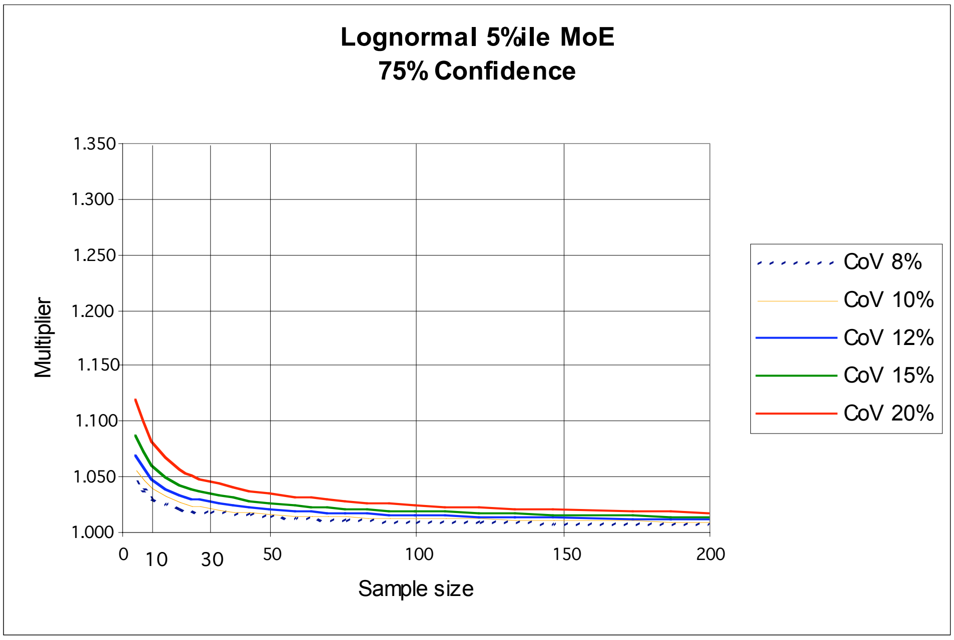
CL	*Value of A*
95%	-2.682
90%	-2.121
85%	-1.737
80%	-1.438
75%	-1.178


Lognormal 5%ile MoE
95% Confidence
Multiplier
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Lognormal 5%ile MoE
90% Confidence
Multiplier
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Lognormal 5%ile MoE
85% Confidence
Multiplier
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Lognormal 5%ile MoE
80% Confidence
Multiplier
1.350
1.300
1.250
1.200
1.150
1.100
1.050
1.000
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%


Lognormal 5%ile MoE
75% Confidence
Multiplier
1.350
1.300
1.250
1.200
1.150
1.100
1.050
1.000
0
10
30
50
100
150
200
Sample size
CoV 8%
CoV 10%
CoV 12%
CoV 15%
CoV 20%

Appendix B – Outputs for Estimating 5%ile Strength

This Appendix presents methods of finding the Test Comparison Value (*TCV*) in order to have given confidence that the production meets or exceeds the Design Value (*DV*).

In estimating the strength (either bending or tension) of all of the pieces in a batch, random samples are taken as representative of the batch.

The strength properties found from testing and analysis of the sampled timber are taken as estimates of the batch properties.

Where the test results are greater than or equal to Test Comparison Values calculated from tables or equations in this Appendix, then the producer has the assigned confidence that the production batch meets or exceeds the Design Value.

The steps in using the appendix are as follows:

1. Decide on a level of confidence that the strength of the material produced meets the Design Value.
2. Select the section appropriate for the analysis method (eg non-parametric, log-normal, log-normal tail fit, 2 parameter Weibull tail fit) to be used.
3. Find the value of Test Comparison Value (*TCV*):
   - For tables, use the selected Confidence Level, sample size and estimated CoV of the grade to look up value of ***M***, and find $TCV = \boldsymbol{M} \times DV$. Interpolation is possible within the table.
   - For equations, use the required Confidence Level to select a value of *A*, and this is used to find the Test Comparison Value for any value of CoV or sample size.
   - For graphs, use the selected Confidence Level, sample size and estimated CoV of the grade to look up value of ***M***, and find $TCV = \boldsymbol{M} \times DV$. (It is more accurate to use either tables or equations, but the graphs present a visual representation of trends.
4. Use the Test Comparison Value in the monitoring system.

Refer to examples in Section 6 for further guidance.

B.1 Multipliers (M) for Non-parametric 5%ile Strength

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

Non-parametric 5%ile Strength								
CL 95%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.090	1.198	1.330	1.494	1.705	1.985	2.374	2.954
10	1.062	1.132	1.213	1.305	1.413	1.540	1.693	1.879
20	1.043	1.090	1.142	1.198	1.261	1.330	1.407	1.494
30	1.035	1.072	1.113	1.156	1.203	1.254	1.309	1.370
50	1.027	1.055	1.085	1.117	1.150	1.186	1.224	1.265
100	1.019	1.038	1.059	1.080	1.102	1.125	1.149	1.174
200	1.013	1.027	1.041	1.055	1.070	1.085	1.101	1.117

Non-parametric 5%ile Strength								
CL 90%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.074	1.159	1.260	1.379	1.523	1.701	1.926	2.220
10	1.051	1.108	1.171	1.241	1.321	1.411	1.515	1.636
20	1.036	1.074	1.115	1.159	1.207	1.260	1.317	1.379
30	1.029	1.059	1.092	1.126	1.163	1.202	1.244	1.289
50	1.022	1.045	1.070	1.095	1.122	1.150	1.179	1.210
100	1.016	1.032	1.048	1.065	1.083	1.102	1.120	1.140
200	1.011	1.022	1.034	1.045	1.057	1.070	1.082	1.095

Non-parametric 5%ile Strength								
CL 85%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.063	1.135	1.216	1.311	1.421	1.552	1.709	1.902
10	1.044	1.092	1.144	1.201	1.265	1.336	1.415	1.505
20	1.031	1.063	1.098	1.135	1.174	1.216	1.262	1.311
30	1.025	1.051	1.078	1.107	1.138	1.170	1.204	1.240
50	1.019	1.039	1.060	1.081	1.103	1.127	1.151	1.176
100	1.013	1.027	1.041	1.056	1.071	1.086	1.102	1.119
200	1.009	1.019	1.029	1.039	1.049	1.060	1.070	1.081

Non-parametric 5%ile Strength								
CL 80%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.054	1.115	1.183	1.260	1.348	1.449	1.566	1.704
10	1.038	1.079	1.123	1.171	1.223	1.281	1.343	1.413
20	1.027	1.054	1.084	1.115	1.148	1.183	1.221	1.260
30	1.022	1.044	1.068	1.092	1.118	1.145	1.173	1.203
50	1.017	1.034	1.052	1.070	1.089	1.109	1.129	1.150
100	1.012	1.024	1.036	1.048	1.061	1.074	1.088	1.102
200	1.008	1.017	1.025	1.034	1.043	1.052	1.061	1.070

Non-parametric 5%ile Strength								
CL 75%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.047	1.099	1.157	1.221	1.292	1.372	1.463	1.566
10	1.033	1.068	1.106	1.147	1.190	1.237	1.288	1.343
20	1.023	1.047	1.073	1.099	1.127	1.157	1.188	1.221
30	1.019	1.038	1.059	1.080	1.102	1.124	1.148	1.173
50	1.014	1.029	1.045	1.061	1.077	1.094	1.111	1.129
100	1.010	1.021	1.031	1.042	1.053	1.065	1.076	1.088
200	1.007	1.014	1.022	1.029	1.037	1.045	1.053	1.061

The non-parametric 5%ile can be estimated for test data by ranking the test data, plotting it against its probability and interpolating to find the 5%ile value.

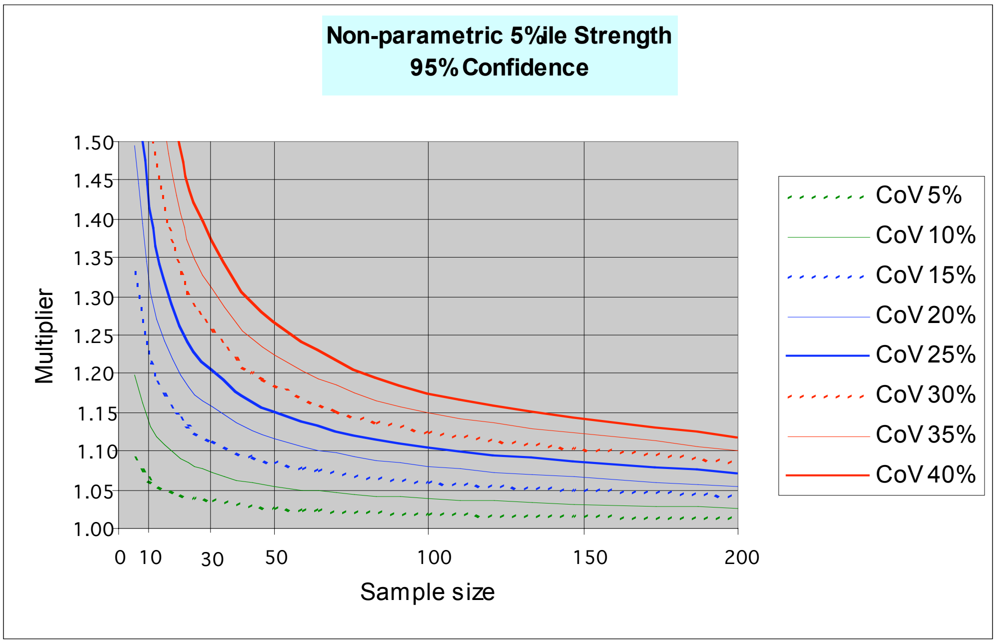
- Detail on the ranking and determination of the 5%ile value is given in Appendix C.2.1.
- The PERCENTILE function in msExcel can be used do this, but has some small errors that may lead to slightly different answers from more rigorously correct analysis methods.

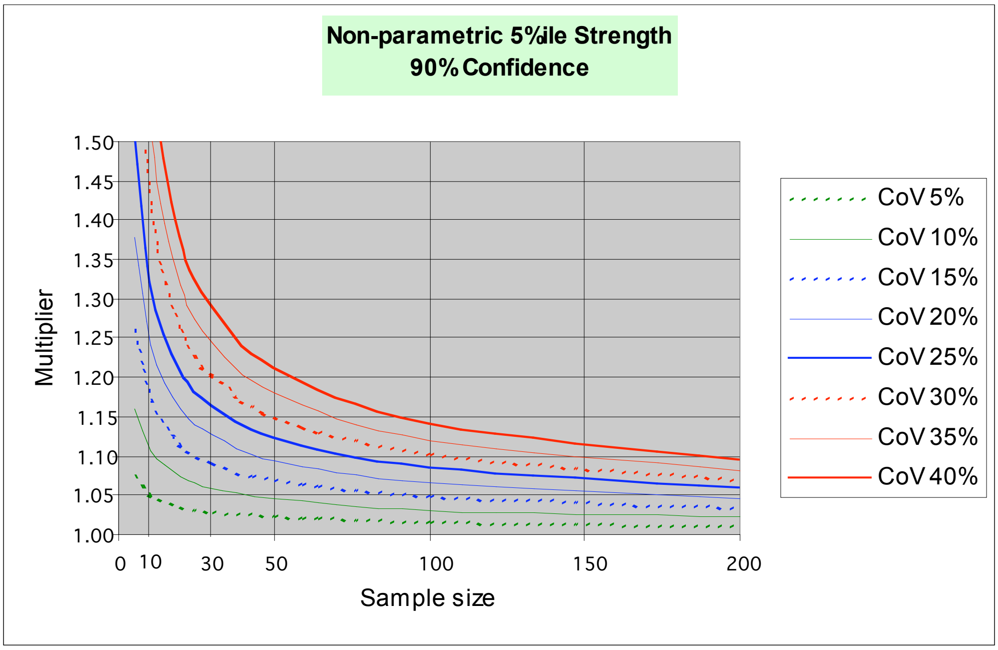
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

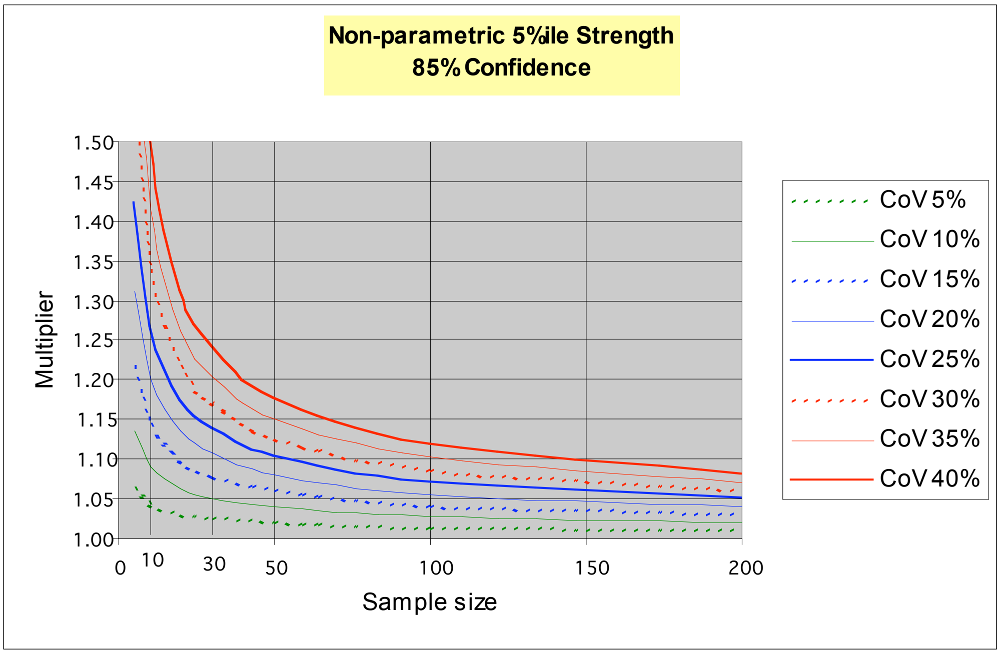
Equation
Non-parametric 5%ile Strength

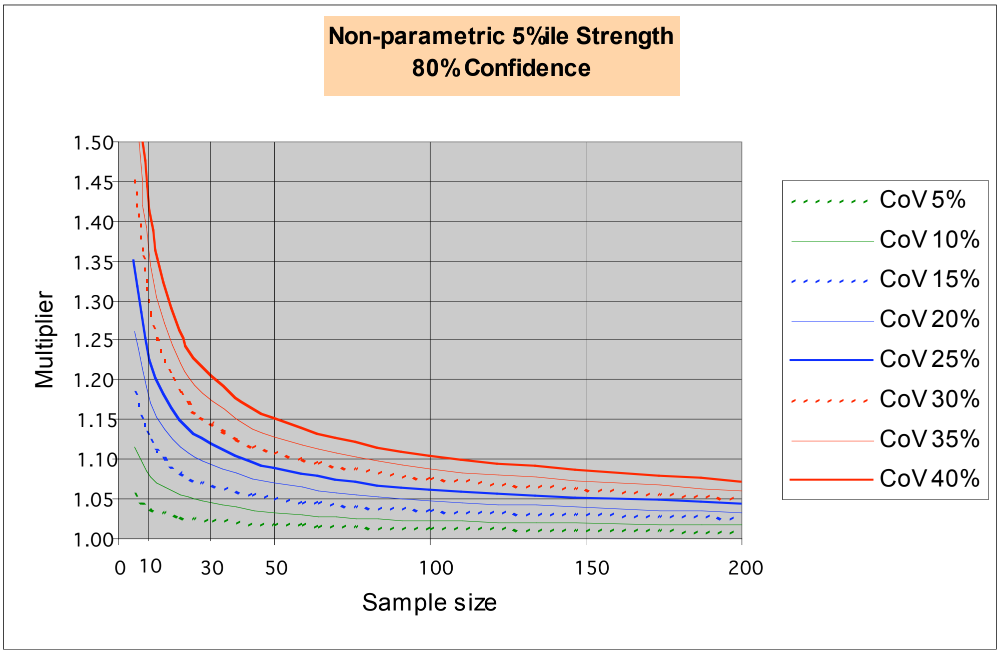
$$TCV = DV\left[\frac{1}{1 + A\dfrac{CoV}{\sqrt{n}}}\right]$$

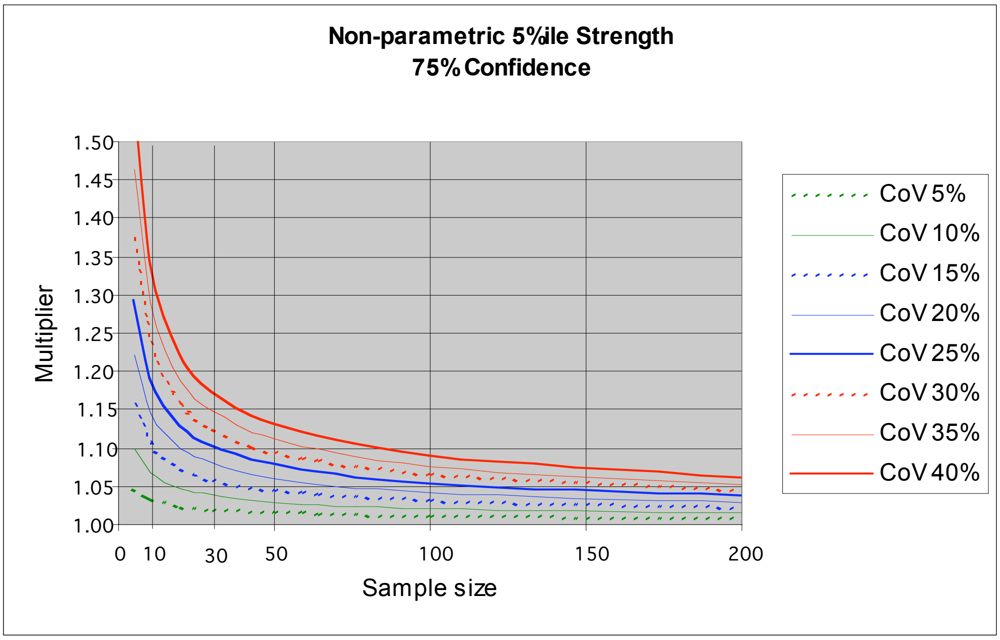
CL	***Value of A***
95%	-3.698
90%	-3.072
85%	-2.651
80%	-2.309
75%	-2.021


Non-parametric 5%ile Strength
95% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Non-parametric 5%ile Strength
90% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Non-parametric 5%ile Strength
85% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Non-parametric 5%ile Strength
80% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Non-parametric 5%ile Strength
75% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%

B.2 Multipliers (M) for Log-normal 5%ile Strength (all data)

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

Log-normal 5%ile Strength (All data)								
CL 95%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.063	1.135	1.217	1.312	1.423	1.554	1.713	1.907
10	1.044	1.092	1.144	1.202	1.266	1.337	1.417	1.507
20	1.031	1.063	1.098	1.135	1.175	1.217	1.263	1.312
30	1.025	1.051	1.079	1.108	1.138	1.170	1.205	1.241
50	1.019	1.039	1.060	1.081	1.104	1.127	1.152	1.177
100	1.013	1.027	1.042	1.056	1.071	1.087	1.103	1.119
200	1.009	1.019	1.029	1.039	1.049	1.060	1.070	1.081

Log-normal 5%ile Strength (All data)								
CL 90%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.049	1.104	1.164	1.232	1.308	1.393	1.491	1.603
10	1.034	1.071	1.111	1.153	1.200	1.249	1.304	1.363
20	1.024	1.049	1.076	1.104	1.133	1.164	1.197	1.232
30	1.020	1.040	1.061	1.083	1.106	1.130	1.155	1.182
50	1.015	1.031	1.047	1.063	1.080	1.098	1.116	1.135
100	1.011	1.021	1.033	1.044	1.056	1.067	1.079	1.092
200	1.007	1.015	1.023	1.031	1.039	1.047	1.055	1.063

Log-normal 5%ile Strength (All data)								
CL 85%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.040	1.084	1.131	1.183	1.240	1.303	1.372	1.449
10	1.028	1.058	1.089	1.123	1.159	1.196	1.237	1.280
20	1.020	1.040	1.062	1.084	1.107	1.131	1.157	1.183
30	1.016	1.033	1.050	1.067	1.086	1.105	1.124	1.145
50	1.012	1.025	1.038	1.051	1.065	1.079	1.094	1.109
100	1.009	1.018	1.027	1.036	1.045	1.055	1.064	1.074
200	1.006	1.012	1.019	1.025	1.032	1.038	1.045	1.051

Log-normal 5%ile Strength (All data)								
CL 80%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.033	1.068	1.106	1.147	1.190	1.238	1.289	1.344
10	1.023	1.047	1.073	1.099	1.128	1.157	1.188	1.221
20	1.016	1.033	1.050	1.068	1.087	1.106	1.126	1.147
30	1.013	1.027	1.041	1.055	1.070	1.085	1.101	1.117
50	1.010	1.021	1.031	1.042	1.053	1.065	1.076	1.088
100	1.007	1.015	1.022	1.029	1.037	1.045	1.053	1.061
200	1.005	1.010	1.015	1.021	1.026	1.031	1.037	1.042

Log-normal 5%ile Strength (All data)								
CL 75%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.027	1.055	1.085	1.117	1.151	1.187	1.225	1.265
10	1.019	1.038	1.059	1.080	1.102	1.125	1.149	1.174
20	1.013	1.027	1.041	1.055	1.070	1.085	1.101	1.117
30	1.011	1.022	1.033	1.045	1.057	1.069	1.081	1.094
50	1.008	1.017	1.025	1.034	1.043	1.052	1.062	1.071
100	1.006	1.012	1.018	1.024	1.030	1.036	1.043	1.049
200	1.004	1.008	1.013	1.017	1.021	1.025	1.030	1.034

The estimation of 5%ile strength using a log-normal distribution through the data makes use of the natural logarithms of the raw strength data (ln(strength)). An extra column needs to be introduced to an analysis spreadsheet.

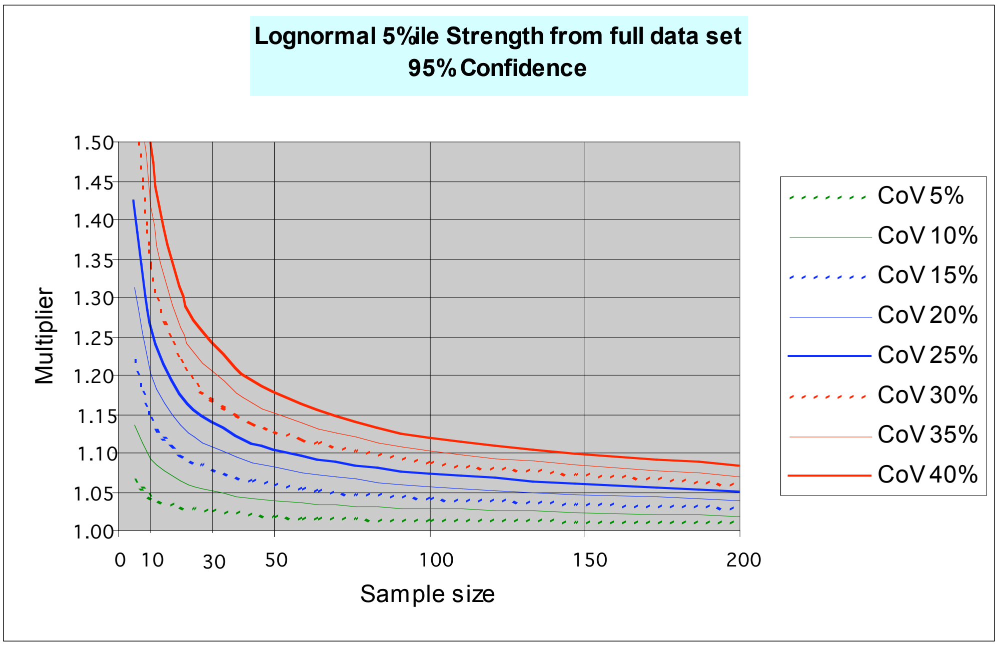
- The estimate of the 5%ile strength is based on the average and the standard deviation of the ln(strength)data.
- The 5%ile strength can be found using (*eqn C.11*).
- The LN, EXP AVERAGE and STDEV functions in msExcel are used.

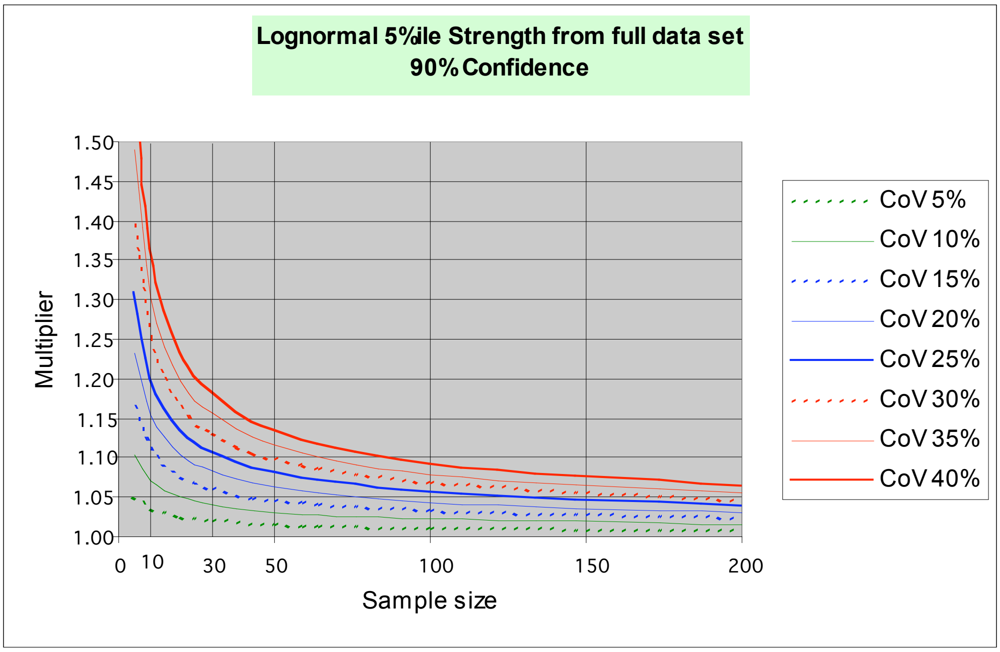
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

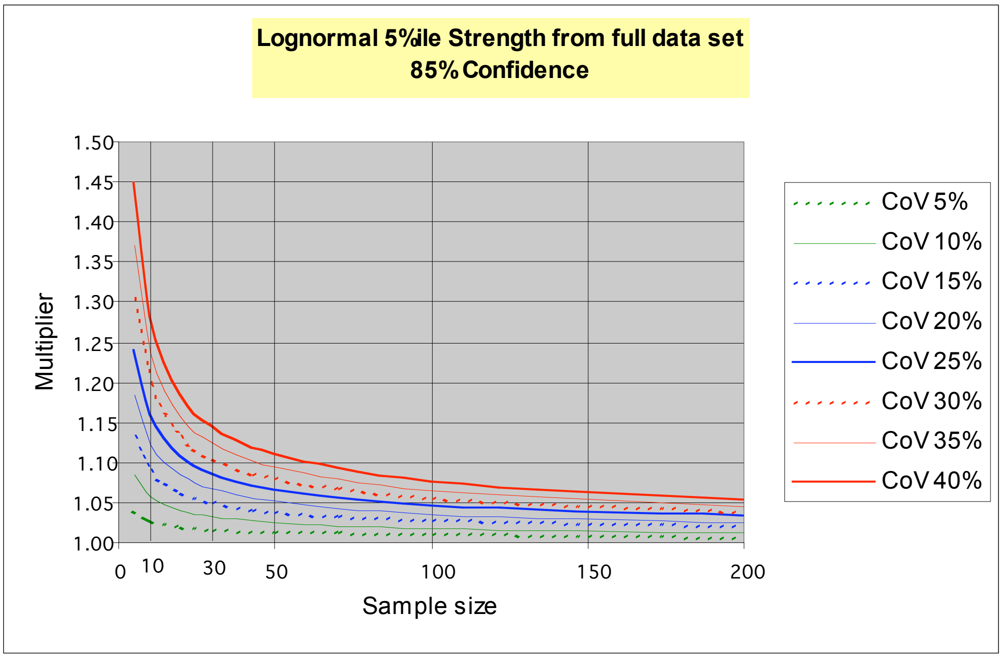
Equation
Log-normal 5%ile Strength (all data)

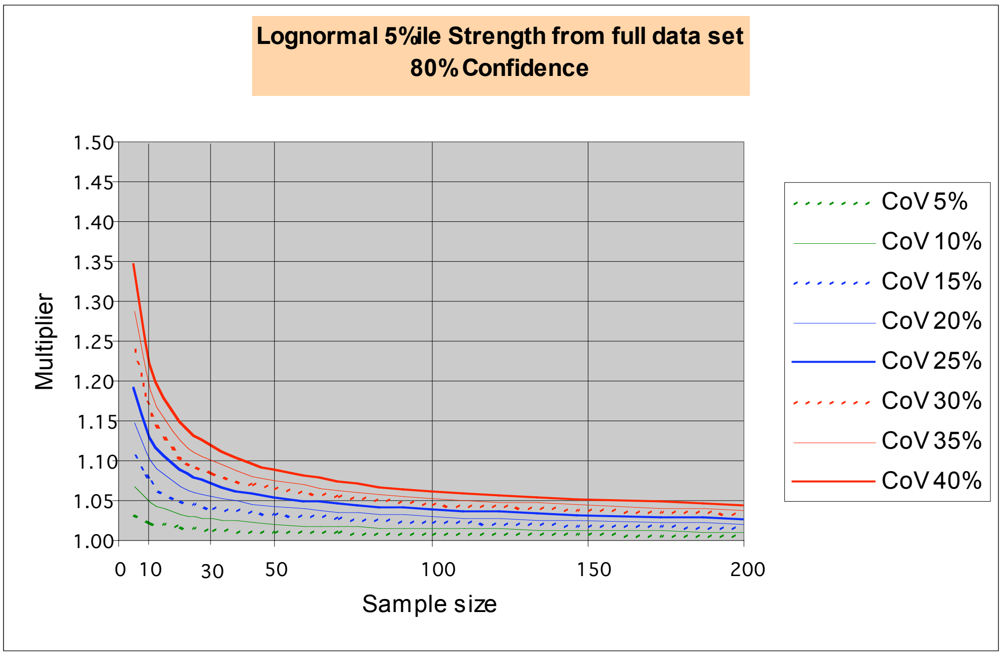
$$TCV = DV\left[\frac{1}{1 + A\frac{CoV}{\sqrt{n}}}\right]$$

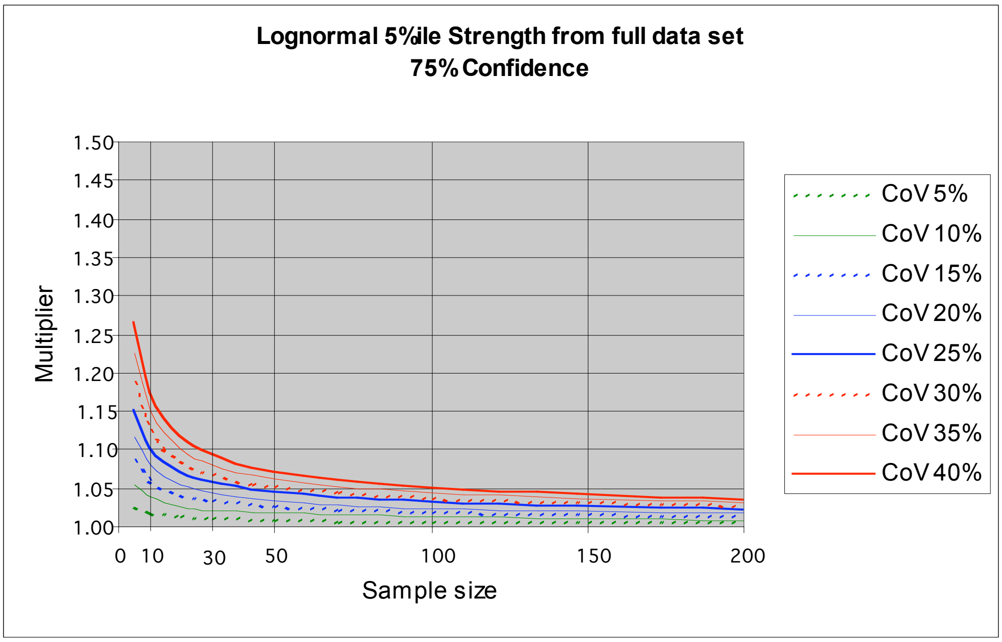
CL	*Value of A*
95%	-2.659
90%	-2.104
85%	-1.731
80%	-1.431
75%	-1.172


Lognormal 5%ile Strength from full data set
95% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from full data set
90% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from full data set
85% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from full data set
80% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from full data set
75% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%

B.3 Multipliers (M) for Log-normal 5%ile Strength from mean (Tight CoV)

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

Log-normal 5%ile Strength (Tight CoV)								
CL 95%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.051	1.107	1.170	1.240	1.319	1.410	1.513	1.632
10	1.035	1.074	1.114	1.159	1.207	1.259	1.315	1.377
20	1.025	1.051	1.078	1.107	1.138	1.170	1.204	1.240
30	1.020	1.041	1.063	1.086	1.110	1.135	1.161	1.188
50	1.016	1.032	1.048	1.065	1.083	1.101	1.120	1.140
100	1.011	1.022	1.034	1.045	1.057	1.069	1.082	1.095
200	1.008	1.016	1.024	1.032	1.040	1.048	1.057	1.065

Log-normal 5%ile Strength (Tight CoV)								
CL 90%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.042	1.088	1.138	1.193	1.253	1.320	1.394	1.477
10	1.029	1.061	1.094	1.129	1.167	1.207	1.250	1.296
20	1.021	1.042	1.064	1.088	1.112	1.138	1.165	1.193
30	1.017	1.034	1.052	1.071	1.090	1.110	1.130	1.152
50	1.013	1.026	1.040	1.054	1.068	1.083	1.098	1.114
100	1.009	1.018	1.028	1.037	1.047	1.057	1.067	1.078
200	1.006	1.013	1.020	1.026	1.033	1.040	1.047	1.054

Log-normal 5%ile Strength (Tight CoV)								
CL 85%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.036	1.075	1.117	1.163	1.212	1.266	1.324	1.389
10	1.025	1.052	1.080	1.110	1.141	1.174	1.209	1.247
20	1.018	1.036	1.055	1.075	1.096	1.117	1.140	1.163
30	1.014	1.029	1.045	1.061	1.077	1.094	1.111	1.129
50	1.011	1.023	1.034	1.046	1.059	1.071	1.084	1.097
100	1.008	1.016	1.024	1.032	1.041	1.049	1.058	1.067
200	1.006	1.011	1.017	1.023	1.028	1.034	1.040	1.046

Log-normal 5%ile Strength (Tight CoV)								
CL 80%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.032	1.065	1.101	1.140	1.181	1.226	1.273	1.325
10	1.022	1.045	1.070	1.095	1.122	1.150	1.179	1.210
20	1.016	1.032	1.048	1.065	1.083	1.101	1.120	1.140
30	1.013	1.026	1.039	1.053	1.067	1.081	1.096	1.111
50	1.010	1.020	1.030	1.040	1.051	1.062	1.073	1.084
100	1.007	1.014	1.021	1.028	1.036	1.043	1.050	1.058
200	1.005	1.010	1.015	1.020	1.025	1.030	1.035	1.040

Log-normal 5%ile Strength (Tight CoV)								
CL 75%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.028	1.057	1.088	1.121	1.156	1.193	1.232	1.275
10	1.019	1.040	1.061	1.082	1.105	1.129	1.154	1.180
20	1.014	1.028	1.042	1.057	1.072	1.088	1.104	1.121
30	1.011	1.022	1.034	1.046	1.058	1.071	1.083	1.096
50	1.009	1.017	1.026	1.035	1.044	1.054	1.063	1.073
100	1.006	1.012	1.018	1.025	1.031	1.037	1.044	1.051
200	1.004	1.009	1.013	1.017	1.022	1.026	1.031	1.035

The estimation of 5%ile strength from the mean using a log-normal distribution through the data makes use of the natural logarithms of the raw strength data (ln(strength)). An extra column needs to be introduced to an analysis spreadsheet.

- The estimate of the 5%ile strength is based on the average of the ln(strength) data.
- *(Eqn C.12)* is used to find the standard deviation to be used in the analysis directly from the long-term CoV of the grade.
- The 5%ile strength can be found using (*eqn C.11*).
- The LN EXP and AVERAGE functions in msExcel are used.

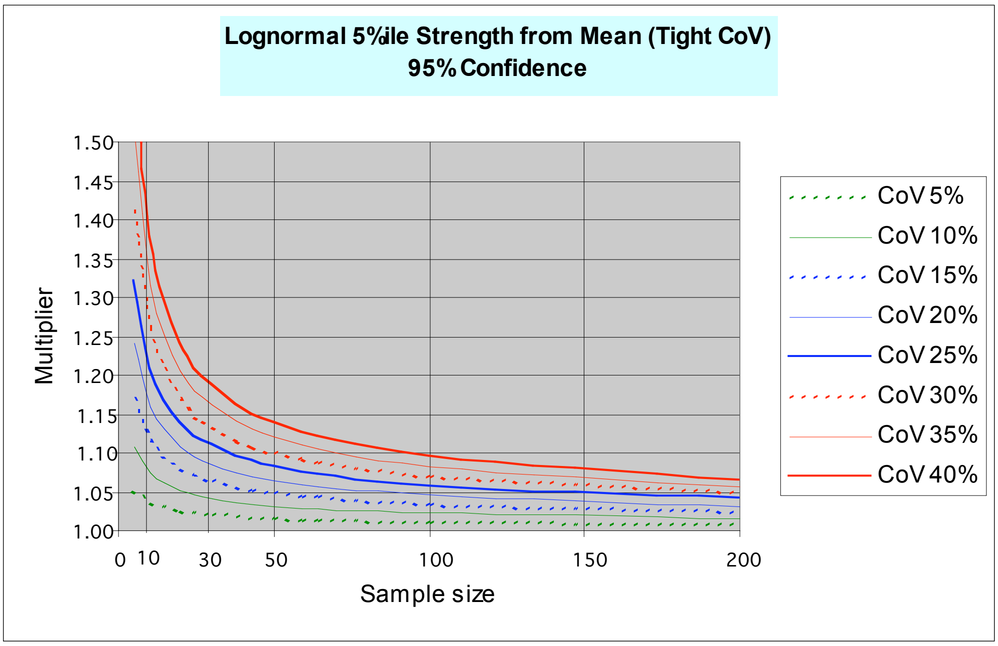
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

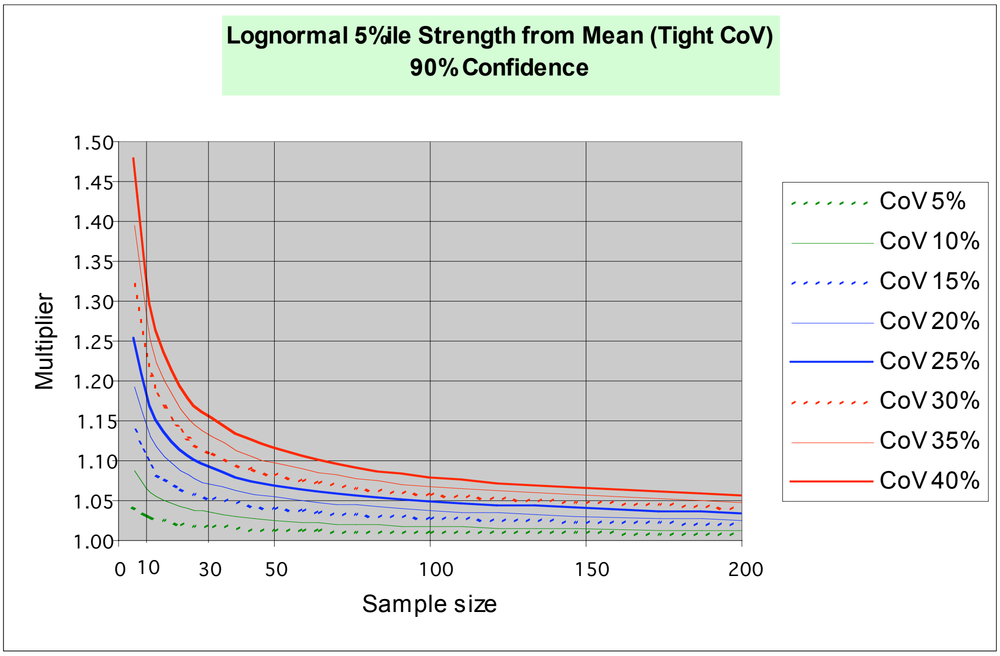
This equation is used when the estimated CoV of production is within 5% of the long-term CoV (ie. Between 0.95 CoV and 1.05 CoV).

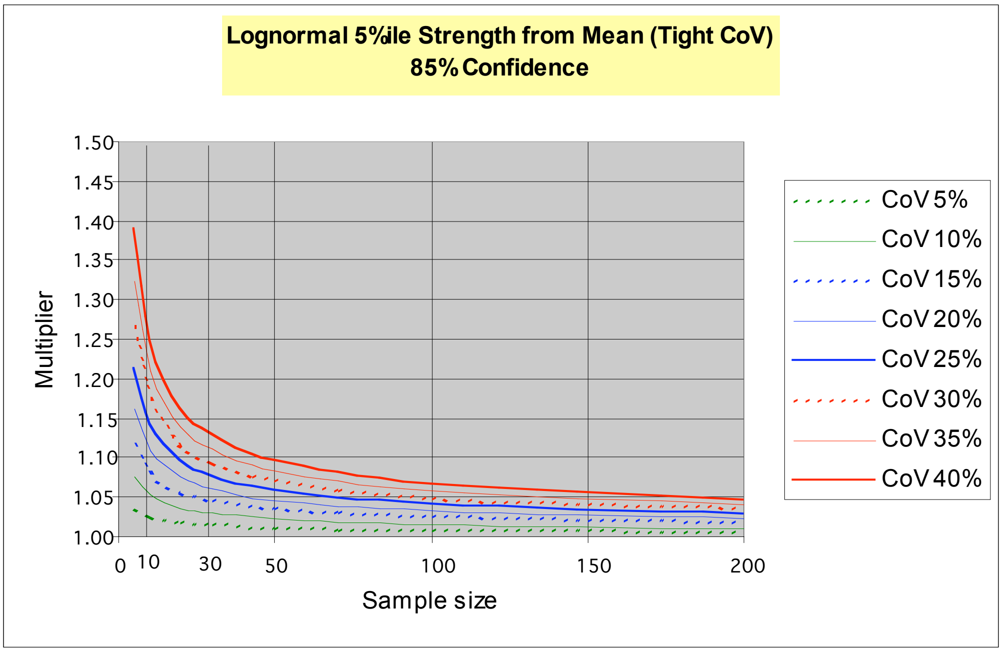
Equation
Log-normal 5%ile strength from mean
(Tight CoV)

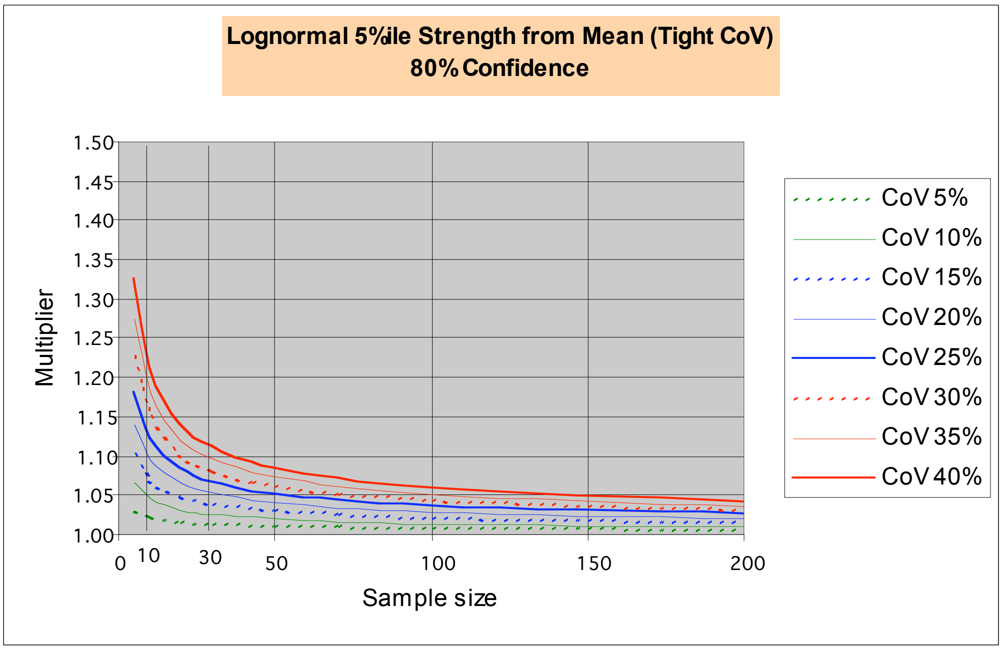
$$TCV = DV\left[\frac{1}{1 + A\frac{CoV}{\sqrt{n}}}\right]$$

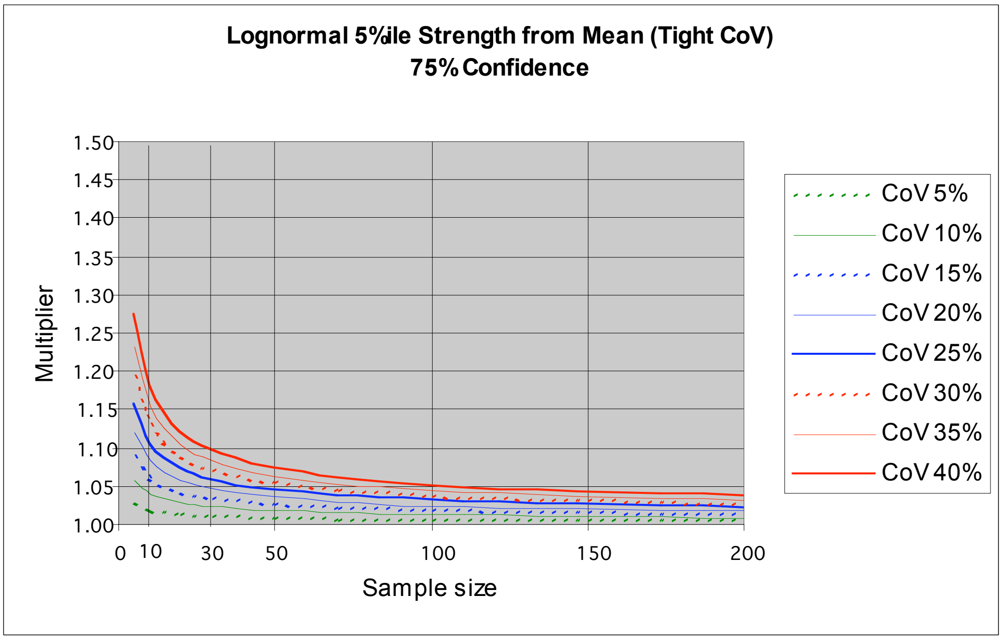
CL	*Value of A*
95%	-2.166
90%	-1.806
85%	-1.564
80%	-1.372
75%	-1.204


Lognormal 5%ile Strength from Mean (Tight CoV)
95% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from Mean (Tight CoV)
90% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from Mean (Tight CoV)
85% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from Mean (Tight CoV)
80% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from Mean (Tight CoV)
75% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%

B.4 Multipliers for Log-normal 5%ile Strength from mean (Loose CoV)

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

Log-normal 5%ile Strength (Loose CoV)								
CL 95%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.064	1.137	1.220	1.317	1.430	1.565	1.728	1.928
10	1.044	1.093	1.146	1.205	1.270	1.343	1.424	1.516
20	1.031	1.064	1.099	1.137	1.177	1.220	1.267	1.317
30	1.025	1.052	1.080	1.109	1.140	1.173	1.208	1.245
50	1.019	1.040	1.061	1.082	1.105	1.129	1.154	1.180
100	1.014	1.028	1.042	1.057	1.072	1.088	1.104	1.121
200	1.010	1.019	1.029	1.040	1.050	1.061	1.071	1.082

Log-normal 5%ile Strength (Loose CoV)								
CL 90%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.055	1.116	1.185	1.263	1.352	1.455	1.574	1.715
10	1.038	1.080	1.124	1.173	1.226	1.284	1.348	1.418
20	1.027	1.055	1.085	1.116	1.150	1.185	1.223	1.263
30	1.022	1.044	1.068	1.093	1.119	1.146	1.175	1.205
50	1.017	1.034	1.052	1.071	1.090	1.110	1.130	1.152
100	1.012	1.024	1.036	1.049	1.062	1.075	1.089	1.103
200	1.008	1.017	1.025	1.034	1.043	1.052	1.061	1.071

Log-normal 5%ile Strength (Loose CoV)								
CL 85%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.049	1.103	1.163	1.230	1.305	1.389	1.486	1.596
10	1.034	1.071	1.110	1.152	1.198	1.247	1.301	1.359
20	1.024	1.049	1.075	1.103	1.132	1.163	1.195	1.230
30	1.019	1.040	1.061	1.083	1.105	1.129	1.154	1.180
50	1.015	1.030	1.046	1.063	1.080	1.097	1.115	1.134
100	1.011	1.021	1.032	1.044	1.055	1.067	1.079	1.091
200	1.007	1.015	1.023	1.030	1.038	1.046	1.055	1.063

Log-normal 5%ile Strength (Loose CoV)								
CL 80%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.044	1.093	1.146	1.204	1.269	1.341	1.422	1.513
10	1.031	1.064	1.099	1.136	1.176	1.219	1.266	1.315
20	1.022	1.044	1.068	1.093	1.119	1.146	1.174	1.204
30	1.018	1.036	1.055	1.074	1.095	1.116	1.138	1.161
50	1.014	1.028	1.042	1.057	1.072	1.087	1.104	1.120
100	1.010	1.019	1.029	1.039	1.050	1.060	1.071	1.082
200	1.007	1.014	1.021	1.028	1.035	1.042	1.049	1.057

Log-normal 5%ile Strength (Loose CoV)								
CL 75%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.040	1.084	1.131	1.183	1.239	1.302	1.371	1.447
10	1.028	1.058	1.089	1.123	1.158	1.196	1.236	1.280
20	1.020	1.040	1.062	1.084	1.107	1.131	1.156	1.183
30	1.016	1.033	1.050	1.067	1.086	1.105	1.124	1.144
50	1.012	1.025	1.038	1.051	1.065	1.079	1.094	1.108
100	1.009	1.018	1.027	1.036	1.045	1.055	1.064	1.074
200	1.006	1.012	1.019	1.025	1.032	1.038	1.045	1.051

The estimation of 5%ile strength from the mean using a log-normal distribution through the data makes use of the natural logarithms of the raw strength data (ln(strength)). An extra column needs to be introduced to an analysis spreadsheet.

- The estimate of the 5%ile strength is based on the average of the ln(strength) data.
- *(Eqn C.12)* is used to find the standard deviation to be used in the analysis directly from the long-term CoV of the grade.
- The 5%ile strength can be found using (*eqn C.11*).
- The LN,EXP and AVERAGE functions in msExcel are used.

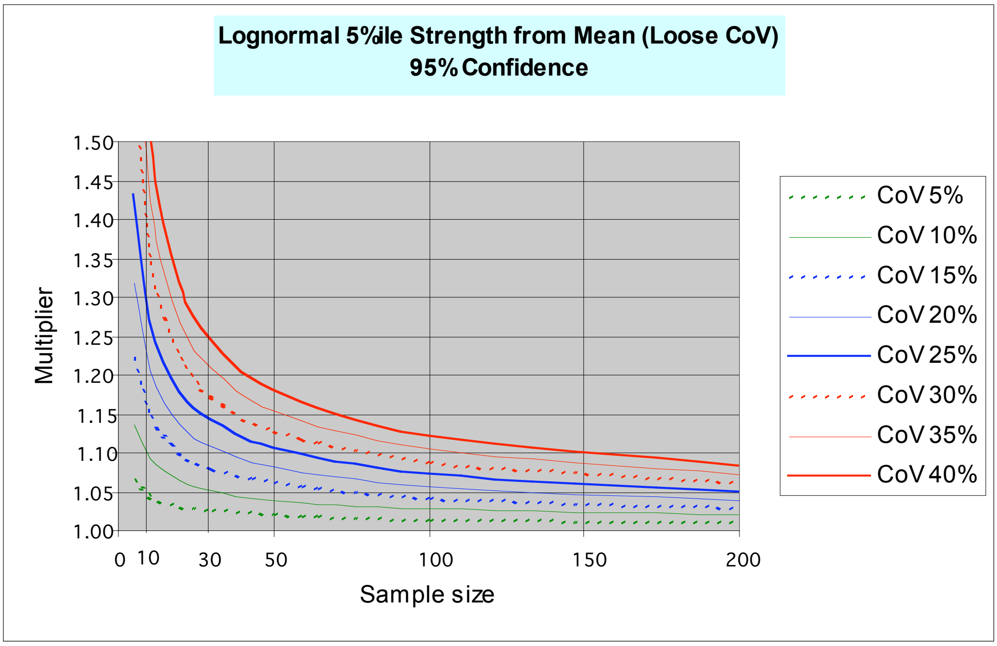
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

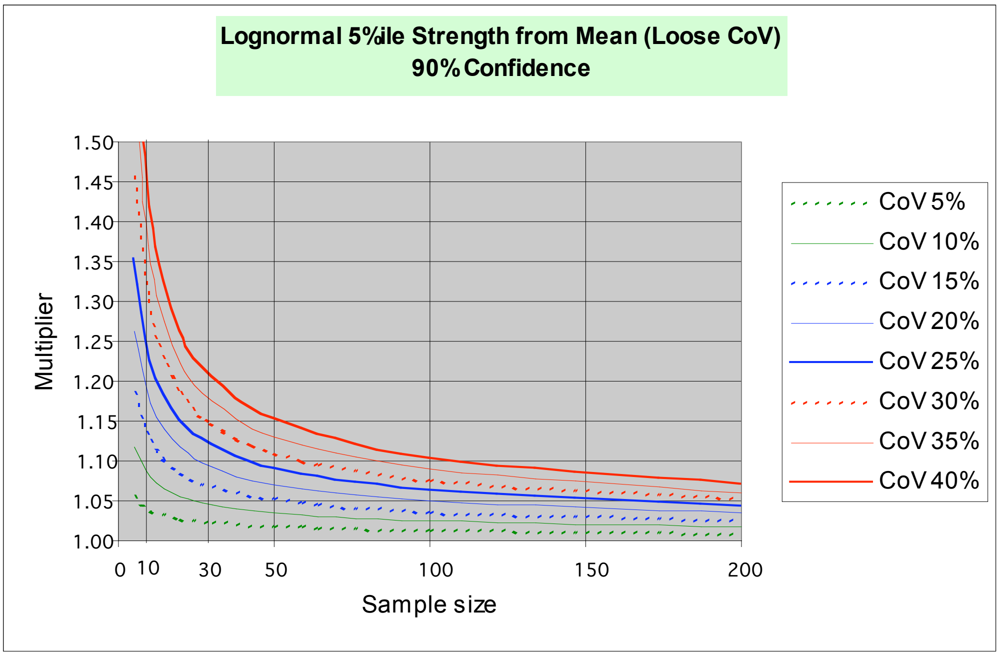
This equation is used when the estimated CoV of production is within 10% of the long-term CoV (ie. Between 0.90 CoV and 1.10 CoV).

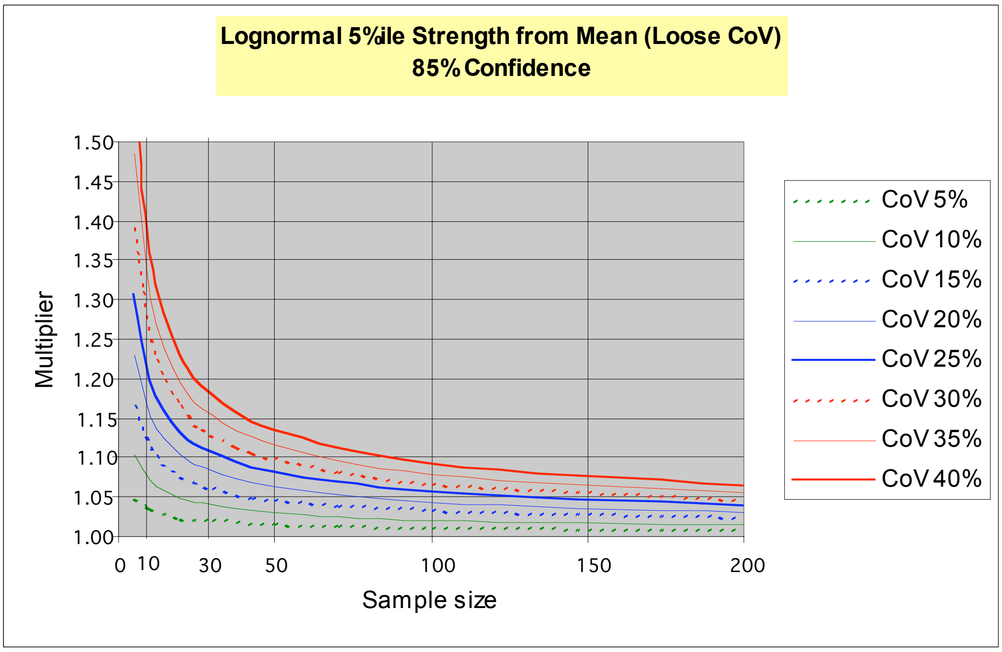
Equation
Log-normal 5%ile strength from mean
(Loose CoV)

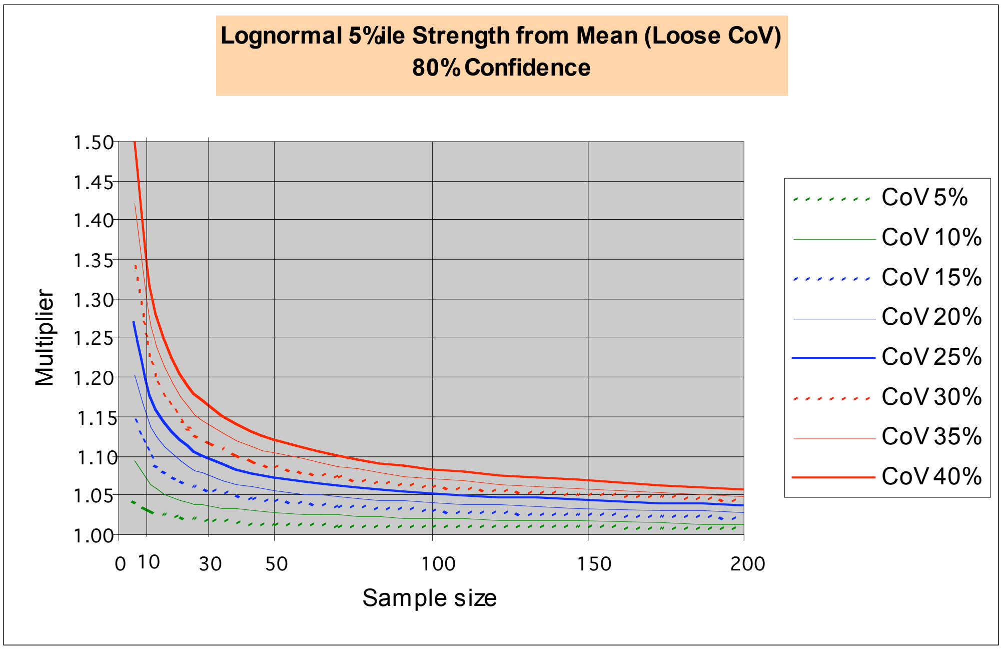
$$TCV = DV\left[\frac{1}{1 + A\frac{CoV}{\sqrt{n}}}\right]$$

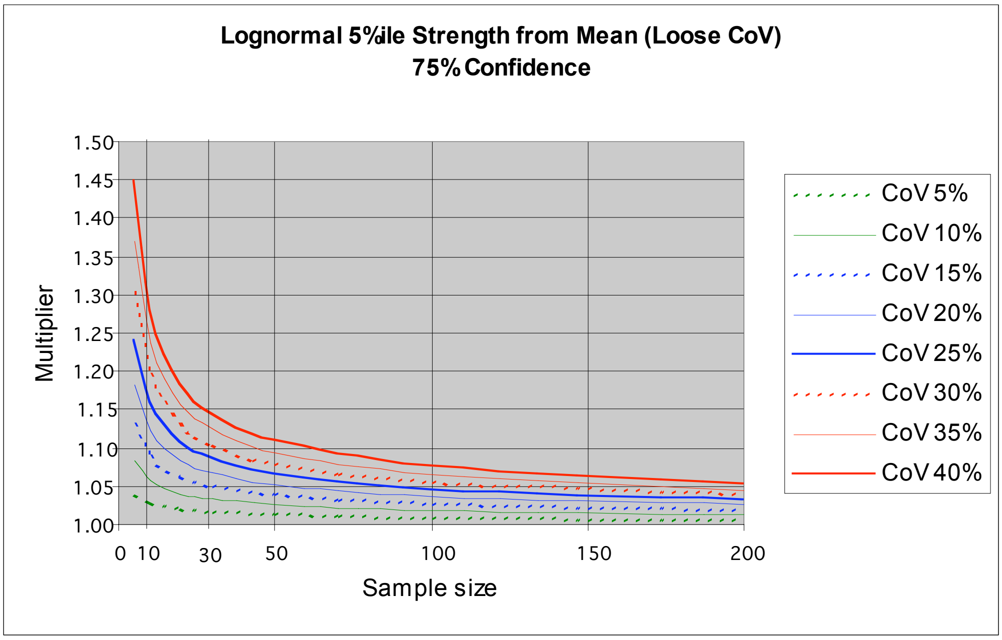
CL	*Value of A*
95%	-2.691
90%	-2.331
85%	-2.089
80%	-1.896
75%	-1.728


Lognormal 5%ile Strength from Mean (Loose CoV)
95% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from Mean (Loose CoV)
90% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from Mean (Loose CoV)
85% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from Mean (Loose CoV)
80% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal 5%ile Strength from Mean (Loose CoV)
75% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%

B.5 Multipliers for Log-normal 5%ile Strength (Tail fit)

$$TCV = \boldsymbol{M} * DV$$

Tables give values of ***M***

Log-normal 5%ile Strength (Tail fit)								
CL 95%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.071	1.154	1.250	1.363	1.499	1.665	1.873	2.139
10	1.049	1.104	1.164	1.232	1.308	1.394	1.491	1.604
20	1.034	1.071	1.111	1.154	1.200	1.250	1.304	1.363
30	1.028	1.057	1.089	1.122	1.157	1.195	1.235	1.278
50	1.022	1.044	1.067	1.092	1.118	1.145	1.173	1.203
100	1.015	1.031	1.047	1.063	1.080	1.098	1.116	1.135
200	1.011	1.022	1.033	1.044	1.056	1.067	1.080	1.092

Log-normal 5%ile Strength (Tail fit)								
CL 90%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.056	1.119	1.190	1.271	1.364	1.470	1.596	1.744
10	1.039	1.082	1.128	1.178	1.232	1.292	1.359	1.432
20	1.027	1.056	1.087	1.119	1.154	1.190	1.229	1.271
30	1.022	1.046	1.070	1.095	1.122	1.150	1.180	1.211
50	1.017	1.035	1.053	1.072	1.092	1.113	1.134	1.156
100	1.012	1.024	1.037	1.050	1.063	1.077	1.091	1.105
200	1.009	1.017	1.026	1.035	1.044	1.053	1.063	1.072

Log-normal 5%ile Strength (Tail fit)								
CL 85%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.046	1.097	1.153	1.215	1.284	1.362	1.449	1.549
10	1.032	1.067	1.104	1.143	1.186	1.231	1.281	1.334
20	1.023	1.046	1.071	1.097	1.124	1.153	1.183	1.215
30	1.018	1.038	1.057	1.078	1.099	1.122	1.145	1.169
50	1.014	1.029	1.044	1.059	1.075	1.092	1.109	1.126
100	1.010	1.020	1.031	1.041	1.052	1.063	1.074	1.086
200	1.007	1.014	1.021	1.029	1.036	1.044	1.052	1.059

Log-normal 5%ile Strength (Tail fit)								
CL 80%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.039	1.080	1.125	1.174	1.228	1.286	1.351	1.422
10	1.027	1.055	1.085	1.117	1.151	1.187	1.225	1.266
20	1.019	1.039	1.059	1.080	1.102	1.125	1.149	1.174
30	1.015	1.031	1.048	1.064	1.082	1.100	1.119	1.138
50	1.012	1.024	1.036	1.049	1.062	1.076	1.089	1.104
100	1.008	1.017	1.026	1.034	1.043	1.052	1.062	1.071
200	1.006	1.012	1.018	1.024	1.030	1.036	1.043	1.049

Log-normal 5%ile Strength (Tail fit)								
CL 75%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.032	1.066	1.102	1.141	1.183	1.228	1.276	1.329
10	1.022	1.046	1.070	1.096	1.123	1.151	1.181	1.212
20	1.016	1.032	1.049	1.066	1.084	1.102	1.121	1.141
30	1.013	1.026	1.039	1.053	1.067	1.082	1.097	1.112
50	1.010	1.020	1.030	1.041	1.051	1.062	1.073	1.085
100	1.007	1.014	1.021	1.028	1.036	1.043	1.051	1.059
200	1.005	1.010	1.015	1.020	1.025	1.030	1.035	1.041

The estimation of 5%ile strength using a log-normal distribution through the tail of the data makes use of the natural logarithms of the raw strength data (ln(strength)). Extra columns need to be introduced to an analysis spreadsheet, to accommodate ln(strength) and probability.

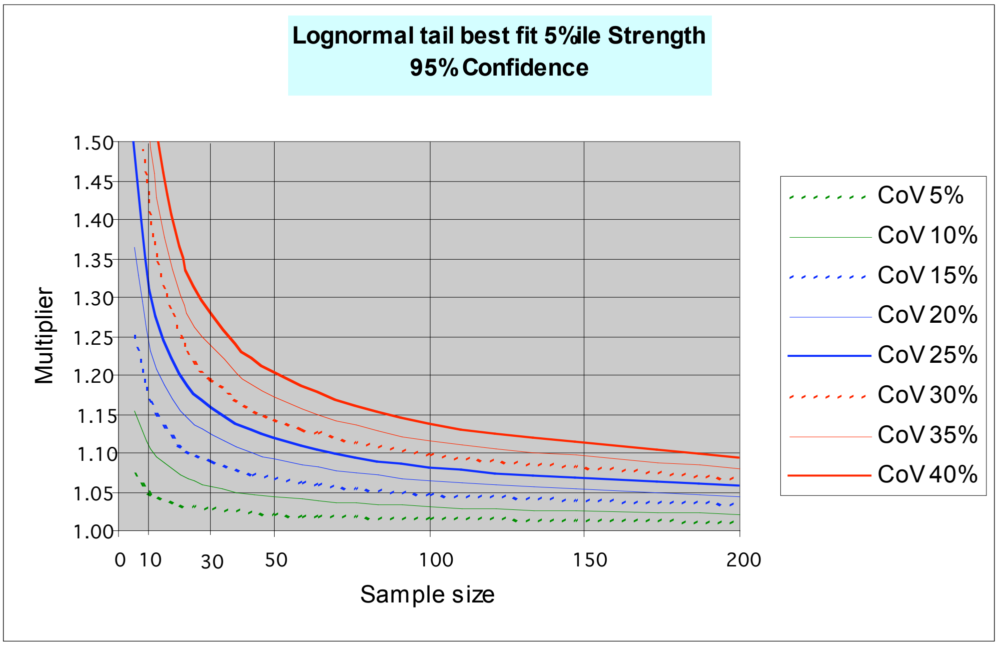
- The data is ranked and probabilities (*pr*) determined in accordance with Appendix C.2.4.
- Ln(strength) is plotted against *z*(*pr*) and the mean and standard deviation of the log-normal distribution are given by the intercept and the slope of the line respectively.
- The 5%ile strength can be found using (*eqn C.11*).
- The LN, EXP, SLOPE and INTERCEPT functions in msExcel are used.

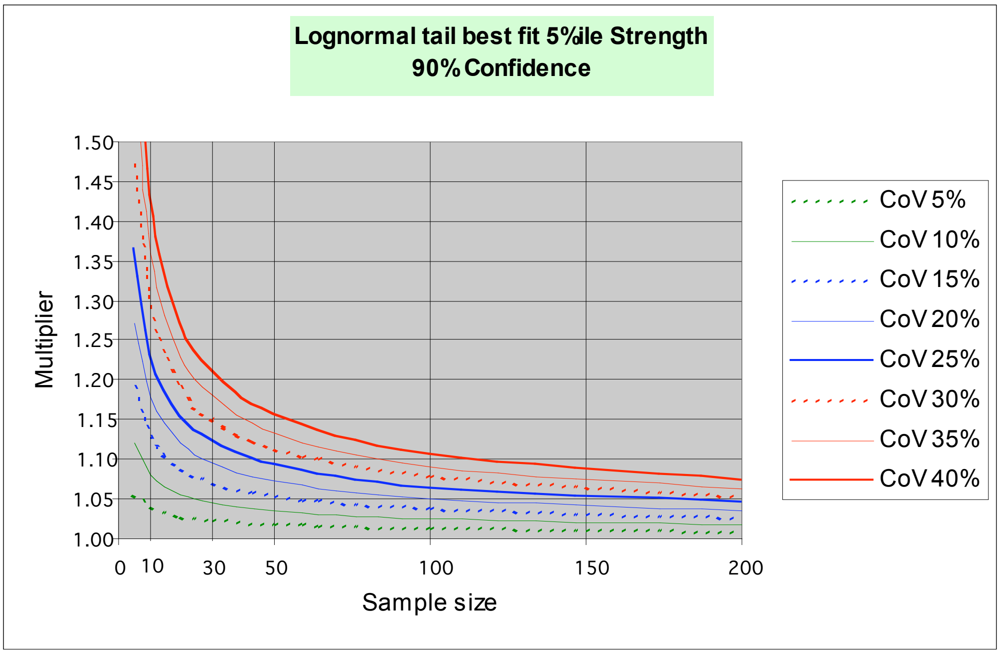
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

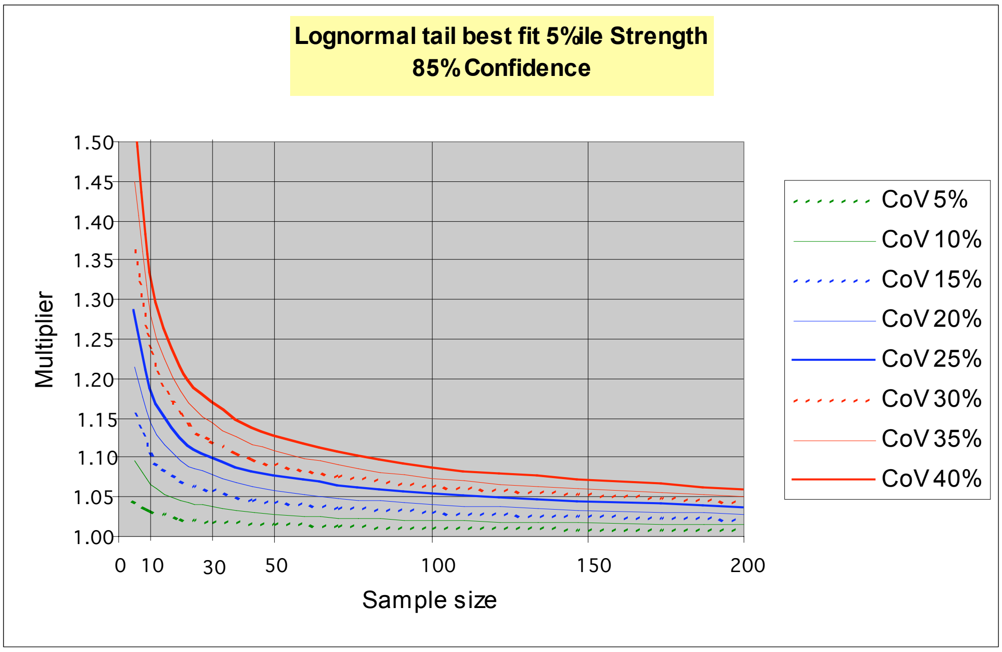
Equation
Log-normal 5%ile Strength
(Tail fit)

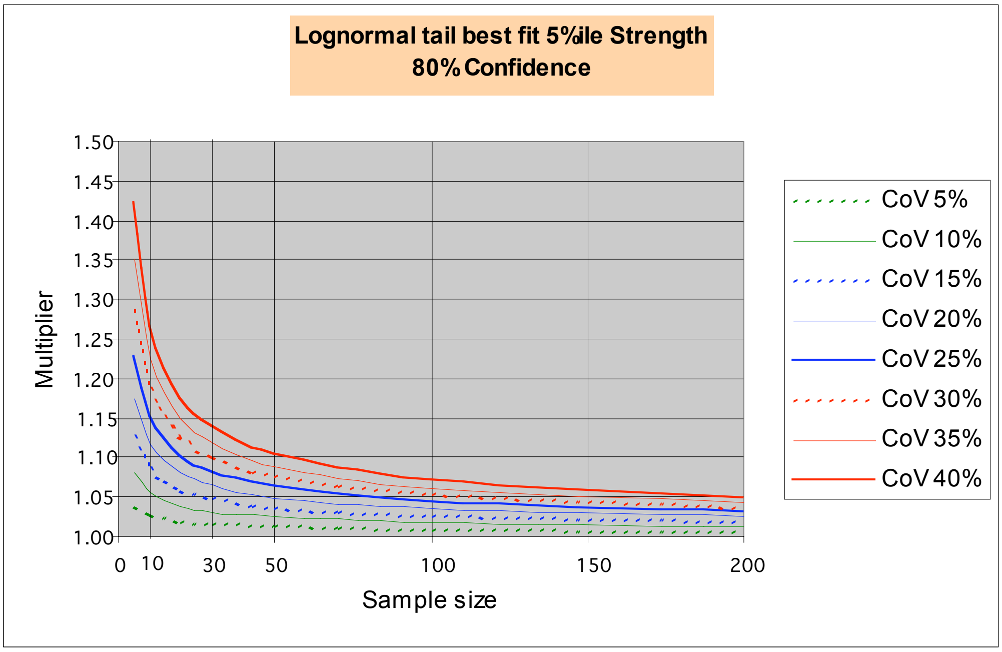
$$TCV = DV\left[\frac{1}{1 + A\frac{CoV}{\sqrt{n}}}\right]$$

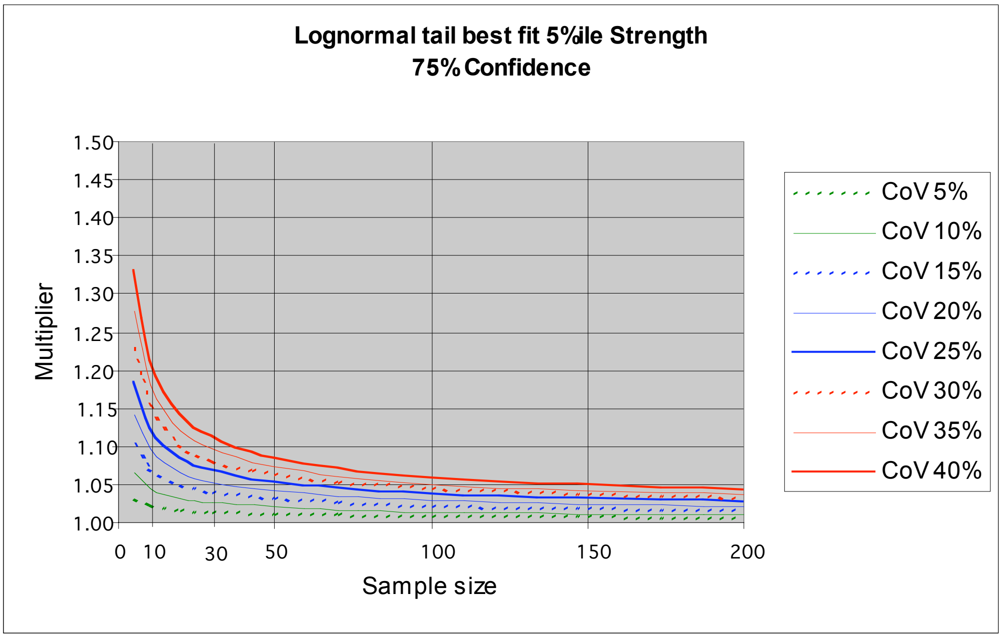
CL	*Value of A*
95%	-2.977
90%	-2.385
85%	-1.981
80%	-1.659
75%	-1.383


Lognormal tail best fit 5%ile Strength
95% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength
90% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength
85% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength
80% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength
75% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%

B.6 Multipliers for Log-normal 5%ile Strength from mean (Tail fit, Tight CoV)

$$TCV = \boldsymbol{M} * DV$$

Tables give values of ***M***

Log-normal 5%ile Strength (Tail fit, Tight CoV)								
CL 95%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.057	1.122	1.194	1.277	1.372	1.482	1.611	1.765
10	1.040	1.083	1.130	1.181	1.237	1.298	1.366	1.442
20	1.028	1.057	1.088	1.122	1.157	1.194	1.234	1.277
30	1.023	1.046	1.071	1.097	1.124	1.153	1.183	1.215
50	1.017	1.035	1.054	1.074	1.094	1.115	1.136	1.159
100	1.012	1.025	1.038	1.051	1.064	1.078	1.093	1.107
200	1.009	1.017	1.026	1.035	1.045	1.054	1.064	1.074

Log-normal 5%ile Strength (Tail fit, Tight CoV)								
CL 90%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.047	1.098	1.155	1.218	1.288	1.366	1.455	1.556
10	1.033	1.067	1.105	1.145	1.188	1.234	1.284	1.338
20	1.023	1.047	1.072	1.098	1.126	1.155	1.185	1.218
30	1.019	1.038	1.058	1.079	1.100	1.123	1.146	1.171
50	1.014	1.029	1.044	1.060	1.076	1.093	1.110	1.127
100	1.010	1.020	1.031	1.042	1.053	1.064	1.075	1.087
200	1.007	1.014	1.022	1.029	1.037	1.044	1.052	1.060

Log-normal 5%ile Strength (Tail fit, Tight CoV)								
CL 85%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.040	1.083	1.130	1.181	1.237	1.298	1.366	1.442
10	1.028	1.057	1.088	1.122	1.157	1.194	1.234	1.277
20	1.020	1.040	1.061	1.083	1.106	1.130	1.155	1.181
30	1.016	1.032	1.049	1.067	1.085	1.104	1.123	1.143
50	1.012	1.025	1.038	1.051	1.064	1.078	1.093	1.107
100	1.009	1.017	1.026	1.035	1.045	1.054	1.064	1.074
200	1.006	1.012	1.019	1.025	1.031	1.038	1.044	1.051

Log-normal 5%ile Strength (Tail fit, Tight CoV)								
CL 80%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.034	1.071	1.111	1.153	1.199	1.249	1.303	1.362
10	1.024	1.049	1.076	1.104	1.133	1.164	1.197	1.232
20	1.017	1.034	1.052	1.071	1.091	1.111	1.132	1.153
30	1.014	1.028	1.042	1.057	1.073	1.089	1.105	1.122
50	1.011	1.021	1.033	1.044	1.055	1.067	1.079	1.092
100	1.007	1.015	1.023	1.031	1.039	1.047	1.055	1.063
200	1.005	1.011	1.016	1.021	1.027	1.033	1.038	1.044

Log-normal 5%ile Strength (Tail fit, Tight CoV)								
CL 75%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.030	1.062	1.095	1.131	1.170	1.211	1.255	1.302
10	1.021	1.043	1.066	1.089	1.114	1.140	1.168	1.196
20	1.015	1.030	1.045	1.062	1.078	1.095	1.113	1.131
30	1.012	1.024	1.037	1.050	1.063	1.076	1.090	1.105
50	1.009	1.019	1.028	1.038	1.048	1.058	1.069	1.079
100	1.007	1.013	1.020	1.027	1.033	1.040	1.048	1.055
200	1.005	1.009	1.014	1.019	1.023	1.028	1.033	1.038

The estimation of 5%ile strength using a log-normal distribution through the tail of the data makes use of the natural logarithms of the raw strength data (ln(strength)). Extra columns need to be introduced to an analysis spreadsheet, to accommodate ln(strength) and probability.

- The data is ranked and probabilities (*pr*) determined in accordance with Appendix C.2.5.
- The standard deviation of the log-normal data is found from (*eqn C.12*) using the estimated CoV of the product.
- The mean of the log-normal distribution is found from the ln(strength) and *pr* data using (*eqn C.14*)
- The 5%ile strength can be found using (*eqn C.11*).
- The LN, EXP and AVERAGE functions in msExcel are used.

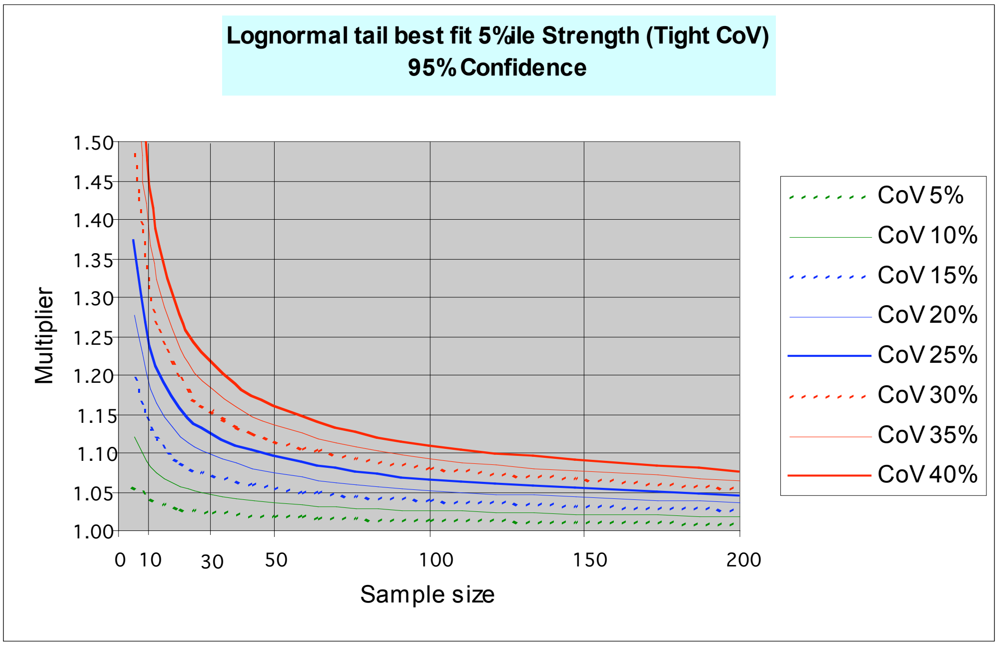
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

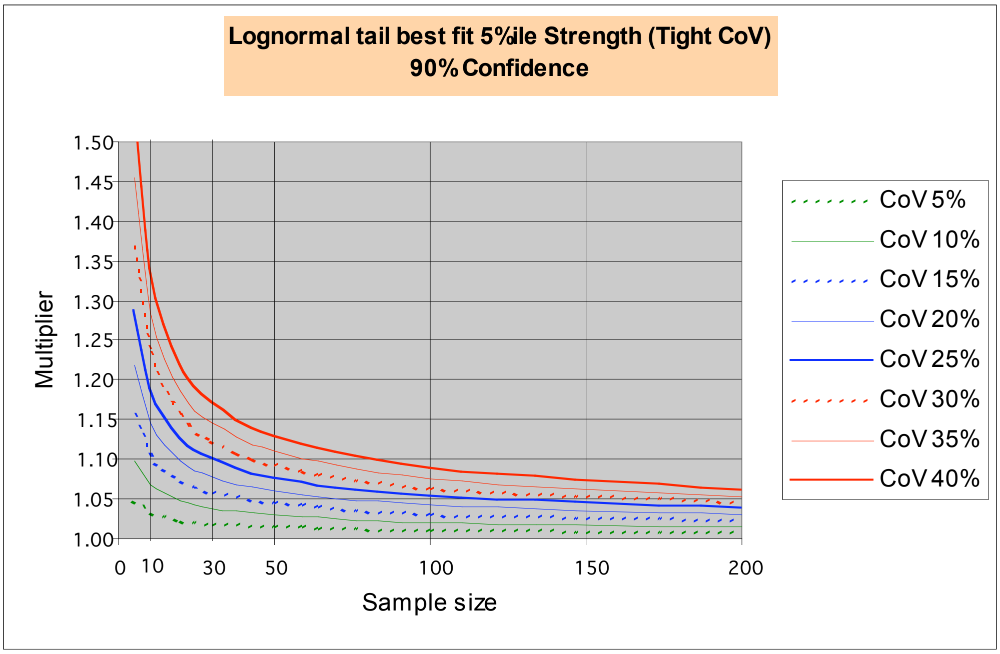
This equation is used when the estimated CoV of production is within 5% of the long-term CoV (ie. Between 0.95 CoV and 1.05 CoV).

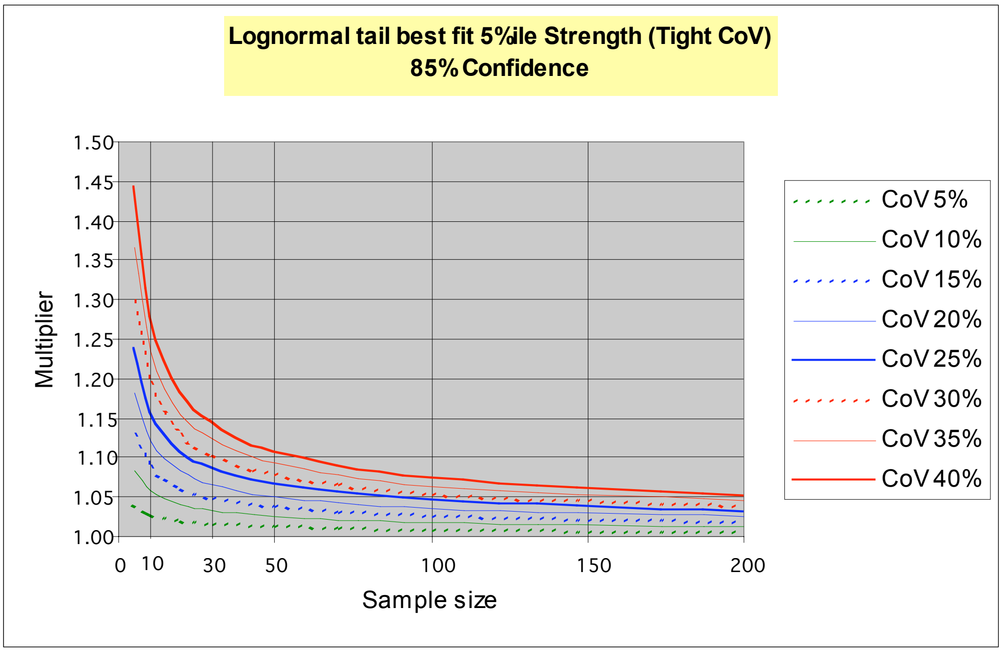
Equation
Log-normal 5%ile Strength from mean
(Tail fit, Tight CoV)

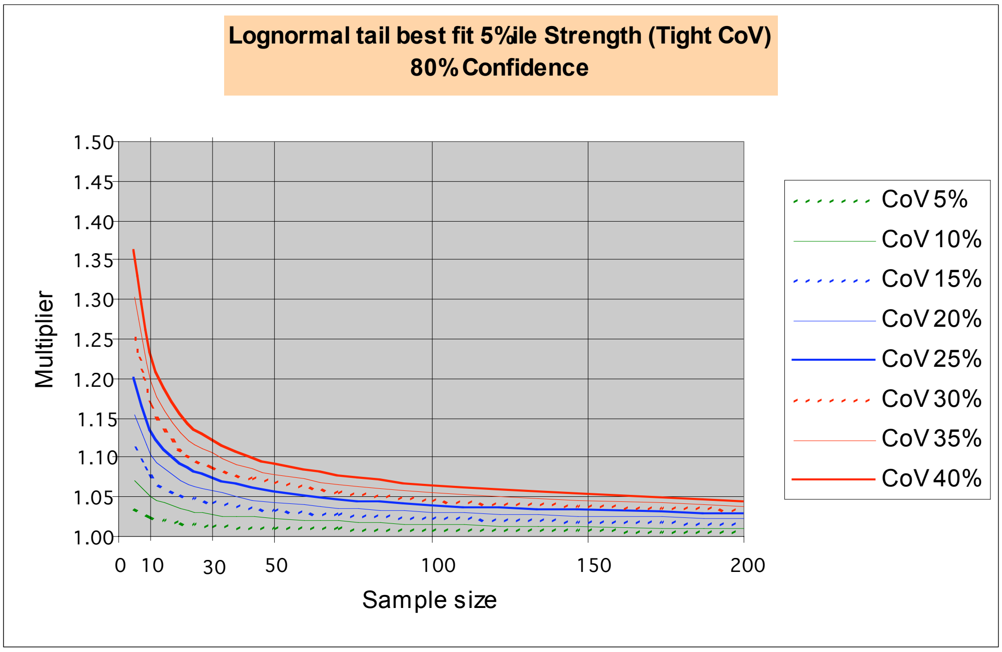
$$TCV = DV\left[\frac{1}{1 + A\dfrac{CoV}{\sqrt{n}}}\right]$$

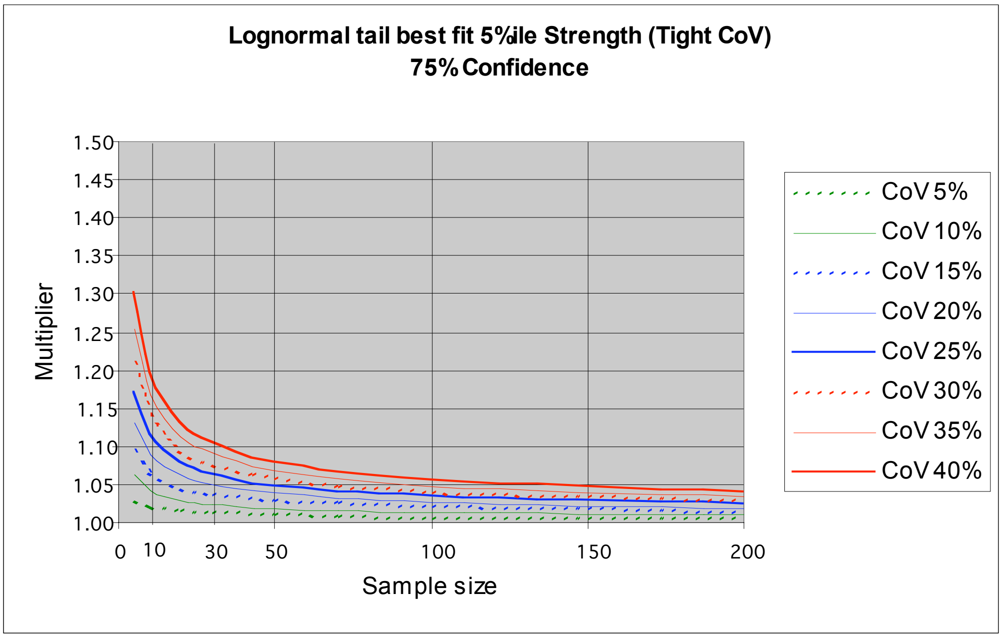
CL	*Value of A*
95%	-2.423
90%	-1.998
85%	-1.713
80%	-1.486
75%	-1.297


Lognormal tail best fit 5%ile Strength (Tight CoV)
95% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength (Tight CoV)
90% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength (Tight CoV)
85% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength (Tight CoV)
80% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength (Tight CoV)
75% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%

B.7 Multipliers for Log-normal 5%ile Strength from mean (Tail fit, Loose CoV)

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

Log-normal 5%ile Strength (Tail fit, Loose CoV)								
CL 95%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.071	1.152	1.247	1.359	1.492	1.655	1.859	2.118
10	1.049	1.103	1.163	1.229	1.304	1.389	1.485	1.596
20	1.034	1.071	1.110	1.152	1.198	1.247	1.300	1.359
30	1.028	1.057	1.088	1.121	1.156	1.193	1.232	1.275
50	1.021	1.044	1.067	1.091	1.116	1.143	1.171	1.200
100	1.015	1.030	1.046	1.063	1.080	1.097	1.115	1.134
200	1.011	1.021	1.032	1.044	1.055	1.067	1.079	1.091

Log-normal 5%ile Strength (Tail fit, Loose CoV)								
CL 90%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.060	1.127	1.204	1.292	1.393	1.512	1.653	1.823
10	1.042	1.087	1.136	1.190	1.249	1.315	1.388	1.469
20	1.029	1.060	1.092	1.127	1.164	1.204	1.246	1.292
30	1.024	1.048	1.074	1.102	1.130	1.160	1.192	1.226
50	1.018	1.037	1.057	1.077	1.098	1.120	1.143	1.167
100	1.013	1.026	1.039	1.053	1.067	1.082	1.097	1.112
200	1.009	1.018	1.028	1.037	1.047	1.057	1.067	1.077

Log-normal 5%ile Strength (Tail fit, Loose CoV)								
CL 85%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.053	1.111	1.177	1.250	1.334	1.429	1.539	1.668
10	1.037	1.076	1.119	1.165	1.215	1.270	1.329	1.395
20	1.026	1.053	1.081	1.111	1.143	1.177	1.212	1.250
30	1.021	1.043	1.065	1.089	1.114	1.140	1.167	1.195
50	1.016	1.033	1.050	1.068	1.086	1.105	1.125	1.145
100	1.011	1.023	1.035	1.047	1.059	1.072	1.085	1.098
200	1.008	1.016	1.024	1.033	1.041	1.050	1.059	1.068

Log-normal 5%ile Strength (Tail fit, Loose CoV)								
CL 80%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.047	1.099	1.156	1.219	1.290	1.370	1.459	1.562
10	1.033	1.068	1.105	1.146	1.189	1.236	1.286	1.341
20	1.023	1.047	1.072	1.099	1.127	1.156	1.187	1.219
30	1.019	1.038	1.058	1.079	1.101	1.124	1.147	1.172
50	1.014	1.029	1.045	1.060	1.077	1.093	1.111	1.128
100	1.010	1.021	1.031	1.042	1.053	1.064	1.076	1.087
200	1.007	1.014	1.022	1.029	1.037	1.045	1.052	1.060

Log-normal 5%ile Strength (Tail fit, Loose CoV)								
CL 75%	**CoV (%)**							
No. Sample	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
5	1.042	1.089	1.139	1.195	1.256	1.323	1.399	1.483
10	1.030	1.061	1.095	1.130	1.168	1.209	1.252	1.299
20	1.021	1.042	1.065	1.089	1.113	1.139	1.166	1.195
30	1.017	1.034	1.052	1.071	1.091	1.111	1.132	1.153
50	1.013	1.026	1.040	1.054	1.069	1.084	1.099	1.115
100	1.009	1.019	1.028	1.038	1.048	1.058	1.068	1.079
200	1.006	1.013	1.020	1.026	1.033	1.040	1.047	1.054

The estimation of 5%ile strength using a log-normal distribution through the tail of the data makes use of the natural logarithms of the raw strength data (ln(strength)). Extra columns need to be introduced to an analysis spreadsheet, to accommodate ln(strength) and probability.

- The data is ranked and probabilities (*pr*) determined in accordance with Appendix C.2.5.
- The standard deviation of the log-normal data is found from (*eqn C.12*) using the estimated CoV of the product.
- The mean of the log-normal distribution is found from the ln(strength) and *pr* data using (*eqn C.14*)
- The 5%ile strength can be found using (*eqn C.11*).
- The LN, EXP and AVERAGE functions in msExcel are used.

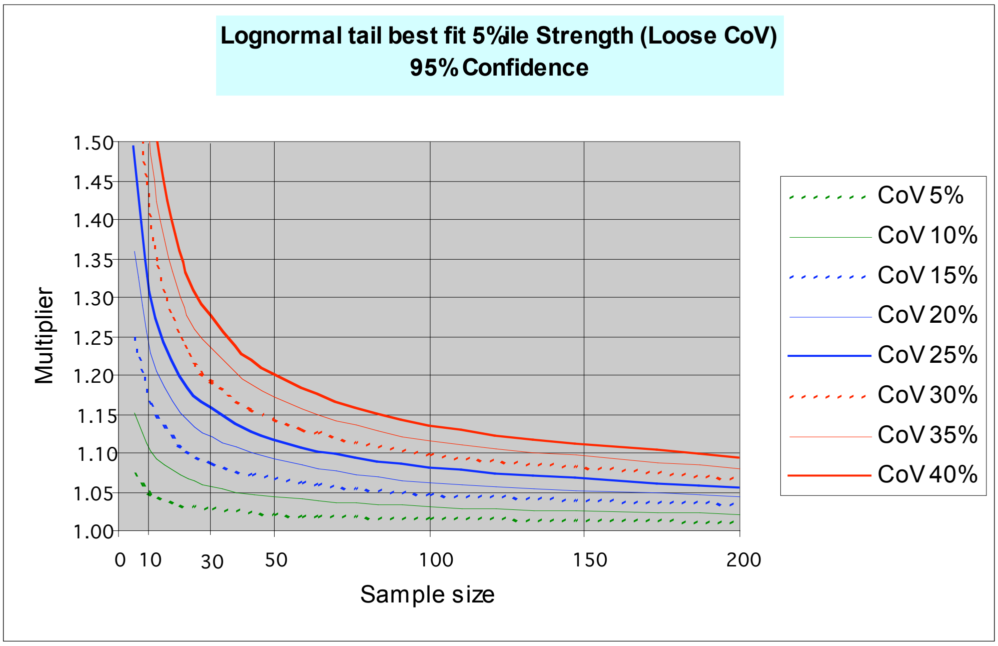
The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

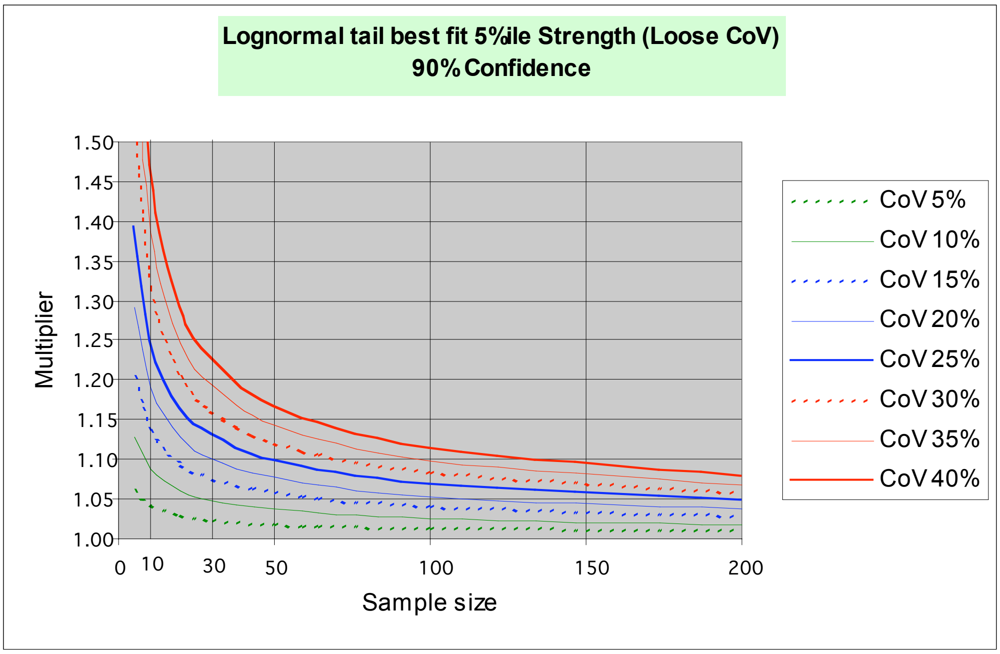
This equation is used when the estimated CoV of production is within 10% of the long-term CoV (ie. Between 0.90 CoV and 1.10 CoV).

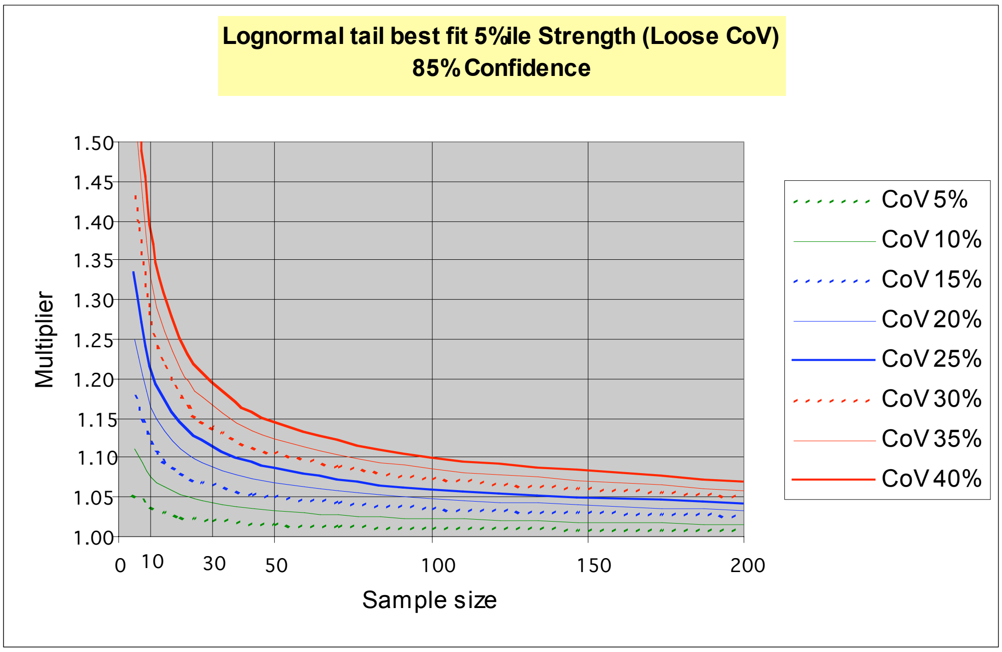
Equation
Log-normal 5%ile Strength from mean
(Tail fit, Loose CoV)

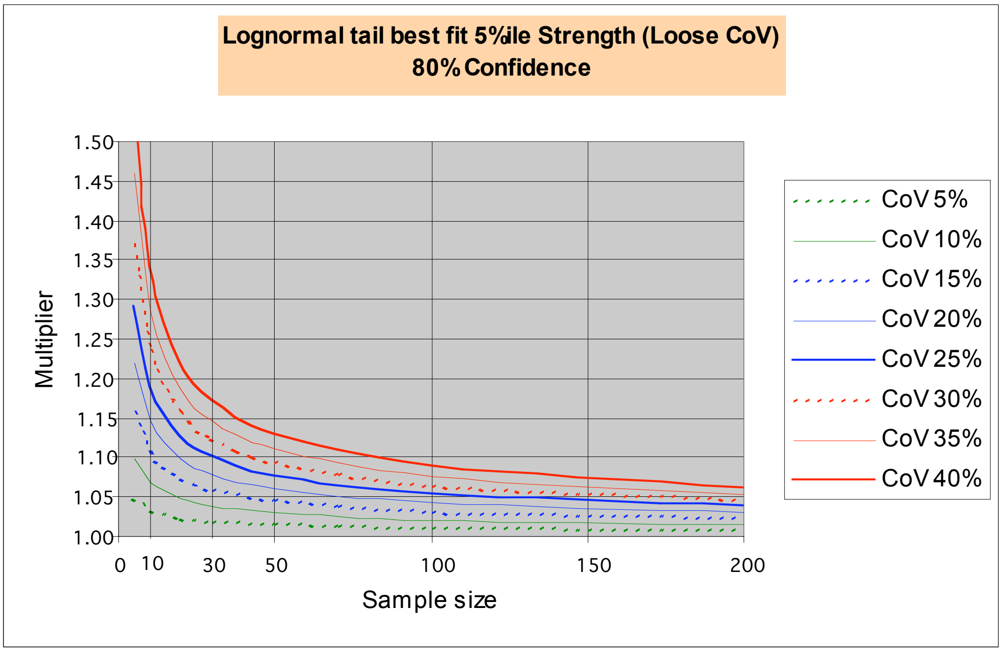
$$TCV = DV\left[\frac{1}{1 + A\frac{CoV}{\sqrt{n}}}\right]$$

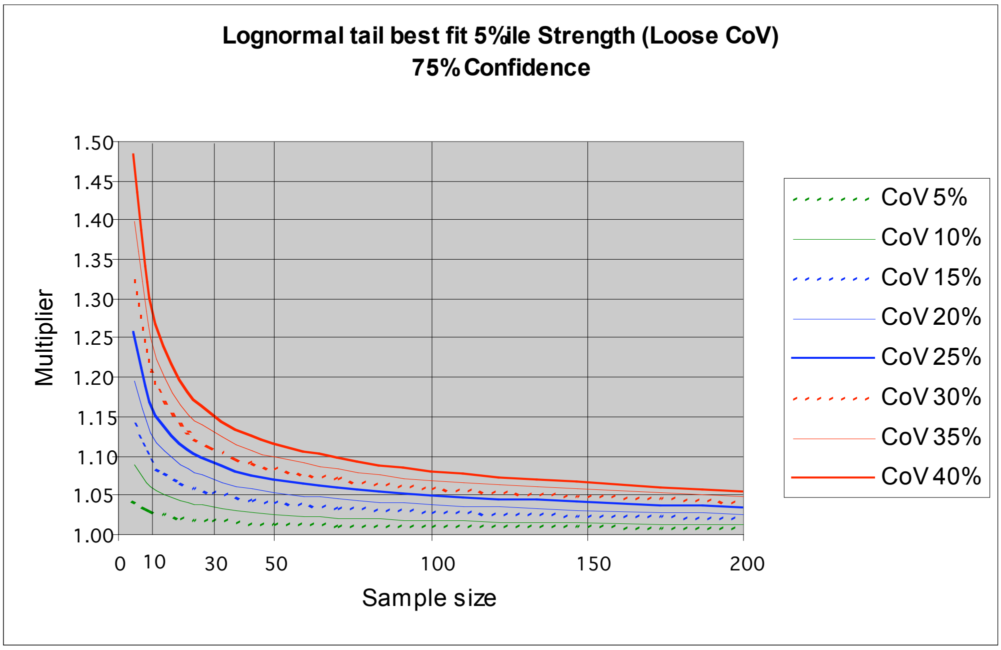
CL	*Value of A*
95%	-2.951
90%	-2.524
85%	-2.239
80%	-2.011
75%	-1.821


Lognormal tail best fit 5%ile Strength (Loose CoV)
95% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength (Loose CoV)
90% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength (Loose CoV)
85% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength (Loose CoV)
80% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


Lognormal tail best fit 5%ile Strength (Loose CoV)
75% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
10
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%

B.8 Multipliers for Two Parameter Weibull (Tail fit) 5%ile Strength

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

2 Parameter Weibull (Tail fit) 5%ile Strength								
CL 95%	**CoV (%)**							
Sample size	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
30	1.061	1.130	1.208	1.298	1.403	1.526	1.673	1.851
50	1.047	1.098	1.154	1.217	1.286	1.364	1.453	1.553
100	1.032	1.067	1.104	1.144	1.187	1.233	1.283	1.337
200	1.023	1.047	1.072	1.098	1.125	1.154	1.185	1.217

2 Parameter Weibull (Tail fit) 5%ile Strength								
CL 90%	**CoV (%)**							
Sample size	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
30	1.049	1.102	1.162	1.228	1.302	1.386	1.481	1.590
50	1.037	1.077	1.121	1.168	1.219	1.275	1.336	1.404
100	1.026	1.054	1.083	1.113	1.146	1.180	1.216	1.255
200	1.018	1.037	1.057	1.077	1.099	1.121	1.144	1.168

2 Parameter Weibull (Tail fit) 5%ile Strength								
CL 85%	**CoV (%)**							
Sample size	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
30	1.041	1.085	1.133	1.186	1.243	1.307	1.377	1.456
50	1.031	1.065	1.100	1.138	1.179	1.222	1.269	1.320
100	1.022	1.045	1.069	1.094	1.120	1.148	1.176	1.207
200	1.015	1.031	1.048	1.065	1.082	1.100	1.119	1.138

2 Parameter Weibull (Tail fit) 5%ile Strength								
CL 80%	**CoV (%)**							
Sample size	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
30	1.034	1.071	1.111	1.153	1.200	1.249	1.304	1.363
50	1.026	1.054	1.084	1.115	1.148	1.183	1.220	1.260
100	1.019	1.038	1.058	1.079	1.100	1.123	1.146	1.171
200	1.013	1.026	1.040	1.054	1.069	1.084	1.099	1.115

2 Parameter Weibull (Tail fit) 5%ile Strength								
CL 75%	**CoV (%)**							
Sample size	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
30	1.029	1.060	1.092	1.127	1.164	1.203	1.245	1.291
50	1.022	1.046	1.070	1.096	1.122	1.150	1.180	1.211
100	1.016	1.032	1.048	1.066	1.084	1.102	1.121	1.141
200	1.011	1.022	1.034	1.046	1.058	1.070	1.083	1.096

The estimation of 5%ile strength using a 2 parameter Weibull distribution through the tail of the data makes use of the natural logarithms of the raw strength data (ln(strength)). Extra columns need to be introduced to an analysis spreadsheet, to accommodate ln(strength) and probability.

- The data is ranked and probabilities (*pr*) determined in accordance with Appendix C.2.6.
- The log(strength) is plotted against a function of *pr* as indicated in Appendix C.2.6.
- The shape and scale parameters can be found from the slope and intercept of the plot.
- The 5%ile strength can be found using (*eqn C.17*).
- The LN, EXP, SLOPE and INTERCEPT functions in msExcel are used.

The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

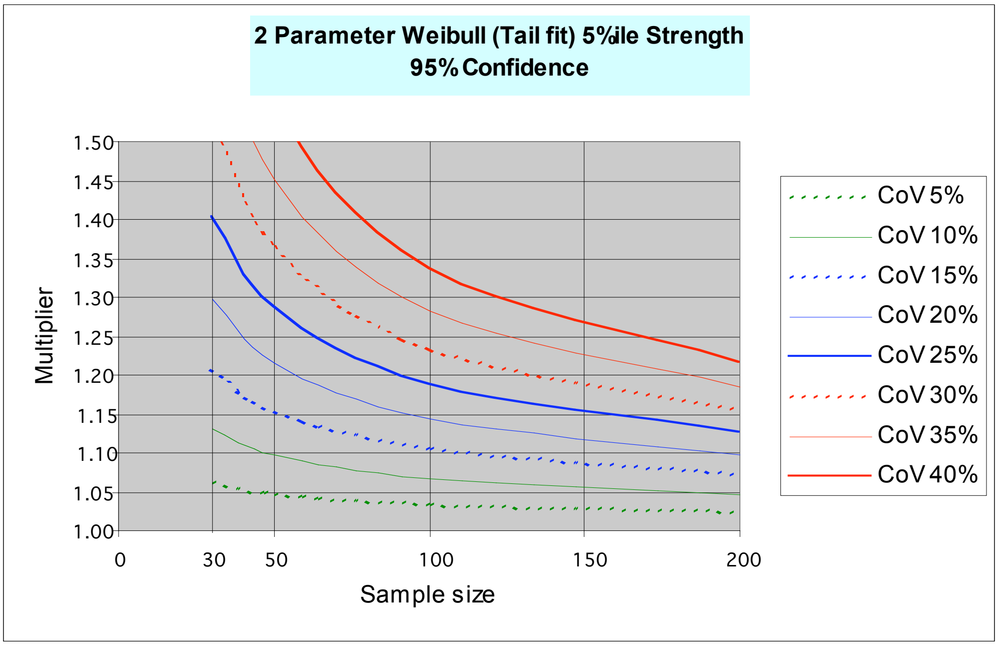
The relationships were established using a proof stress equivalent to the 15%ile of the whole data.

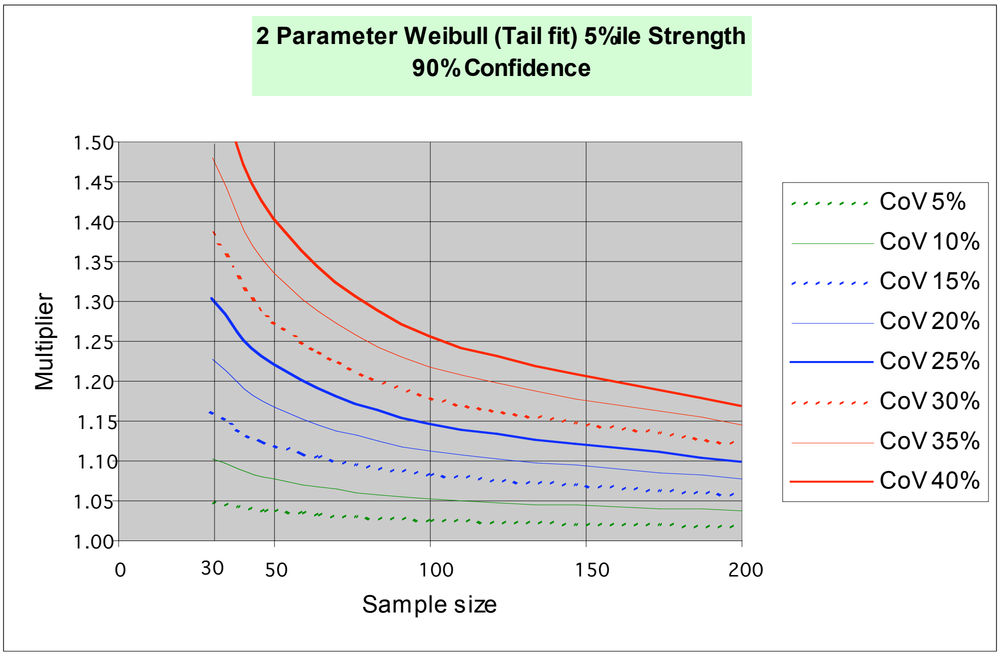
Note: This method is valid for sample sizes greater than 30.

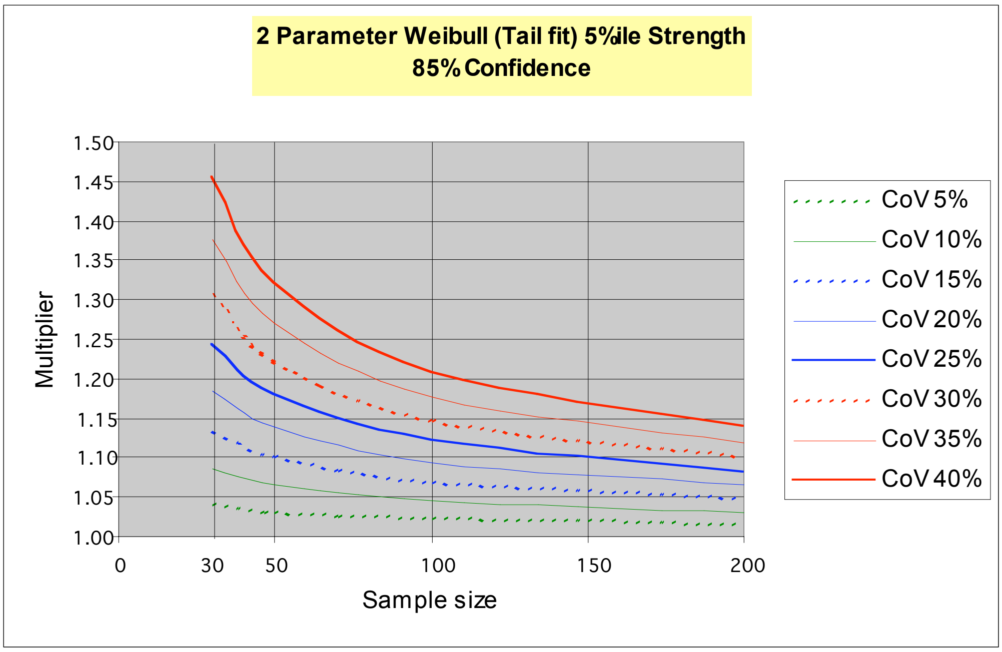
Equation
2 Parameter Weibull (Tail fit)
5%ile Strength

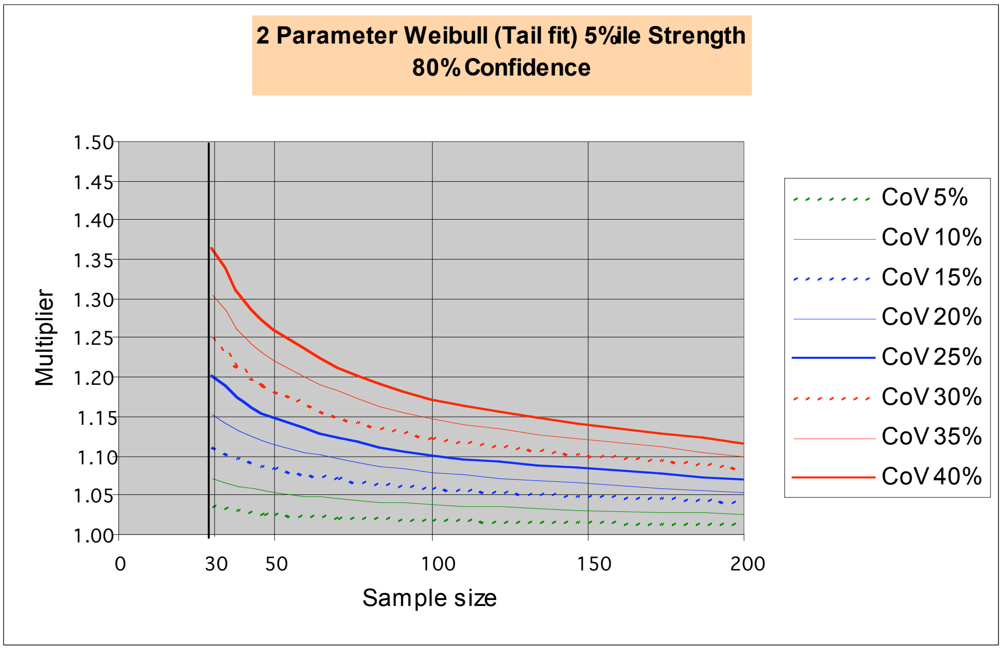
$$TCV = DV\left[\frac{1}{1 + A\frac{CoV}{\sqrt{n}}}\right]$$

CL	*Value of A*
95%	-6.295
90%	-5.084
85%	-4.286
80%	-3.644
75%	-3.083

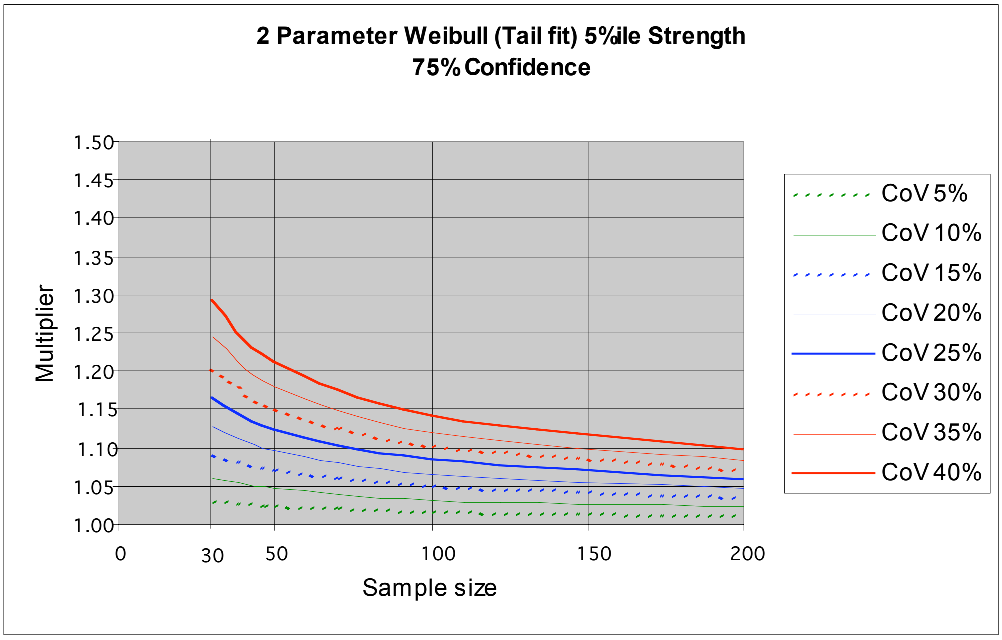

2 Parameter Weibull (Tail fit) 5%ile Strength
95% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


2 Parameter Weibull (Tail fit) 5%ile Strength
90% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


2 Parameter Weibull (Tail fit) 5%ile Strength
85% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%



2 Parameter Weibull (Tail fit) 5%ile Strength
80% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%



2 Parameter Weibull (Tail fit) 5%ile Strength
75% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%

B.9 Multipliers for Two Parameter Weibull (Tail fit) ISO 13910 5%ile Strength

$$\boxed{TCV = \boldsymbol{M} * DV}$$

Tables give values of ***M***

2 Parameter Weibull (Tail fit) ISO 13910 5%ile Strength								
CL 95%	**CoV (%)**							
Sample size	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
30	1.029	1.060	1.093	1.128	1.165	1.205	1.248	1.293
50	1.022	1.046	1.071	1.096	1.123	1.152	1.182	1.213
100	1.016	1.032	1.049	1.066	1.084	1.103	1.122	1.142
200	1.011	1.022	1.034	1.046	1.058	1.071	1.083	1.096

2 Parameter Weibull (Tail fit) ISO 13910 5%ile Strength								
CL 90%	**CoV (%)**							
Sample size	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
30	1.023	1.046	1.071	1.097	1.124	1.153	1.183	1.215
50	1.017	1.035	1.054	1.073	1.094	1.114	1.136	1.158
100	1.012	1.025	1.038	1.051	1.064	1.078	1.092	1.107
200	1.009	1.017	1.026	1.035	1.045	1.054	1.064	1.073

2 Parameter Weibull (Tail fit) ISO 13910 5%ile Strength								
CL 85%	**CoV (%)**							
Sample size	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
30	1.018	1.037	1.056	1.077	1.098	1.120	1.142	1.166
50	1.014	1.028	1.043	1.058	1.074	1.090	1.107	1.124
100	1.010	1.020	1.030	1.041	1.051	1.062	1.073	1.085
200	1.007	1.014	1.021	1.028	1.036	1.043	1.051	1.058

2 Parameter Weibull (Tail fit) ISO 13910 5%ile Strength								
CL 80%	**CoV (%)**							
Sample size	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
30	1.015	1.030	1.045	1.061	1.078	1.095	1.112	1.130
50	1.011	1.023	1.035	1.047	1.059	1.072	1.085	1.098
100	1.008	1.016	1.024	1.033	1.041	1.050	1.058	1.067
200	1.006	1.011	1.017	1.023	1.029	1.035	1.041	1.047

2 Parameter Weibull (Tail fit) ISO 13910 5%ile Strength								
CL 75%	**CoV (%)**							
Sample size	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
30	1.012	1.024	1.036	1.048	1.061	1.074	1.088	1.101
50	1.009	1.018	1.027	1.037	1.047	1.056	1.067	1.077
100	1.006	1.013	1.019	1.026	1.033	1.039	1.046	1.053
200	1.004	1.009	1.014	1.018	1.023	1.027	1.032	1.037

The estimation of 5%ile strength using a 2 parameter Weibull distribution through the tail of the data makes use of the natural logarithms of the raw strength data (ln(strength)). Extra columns need to be introduced to an analysis spreadsheet, to accommodate ln(strength) and probability.

- The data is ranked and probabilities (*pr*) determined in accordance with Appendix C.2.6. The tail of the distribution used in the analysis has the greater of 15 points or 15% of the sample. The lowest two points are discarded in the remainder of the analysis.
- The log(strength) is plotted against a function of *pr* as indicated in Appendix C.2.6.
- The shape and scale parameters can be found from the slope and intercept of the plot.
- The 5%ile strength can be found using (*eqn C.17*).
- The LN, EXP, SLOPE and INTERCEPT functions in msExcel are used.

The Test Comparison Value used to accept or reject this estimate is found using the following equation or the preceding Tables.

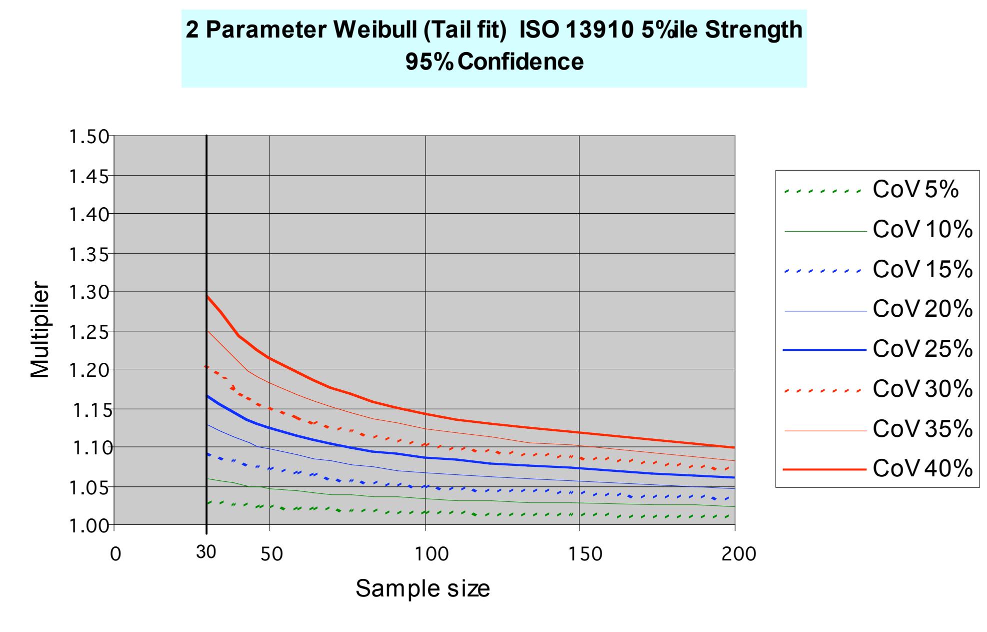
The relationships were established using a proof stress equivalent to the 15%ile of the whole data.

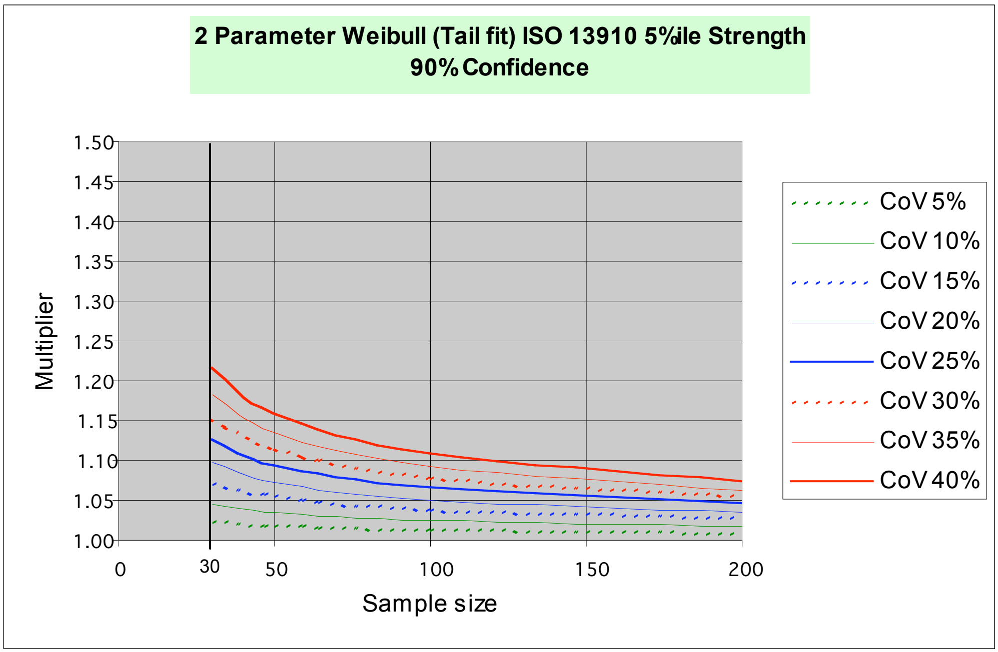
Note: This method is valid for sample sizes greater than 30.

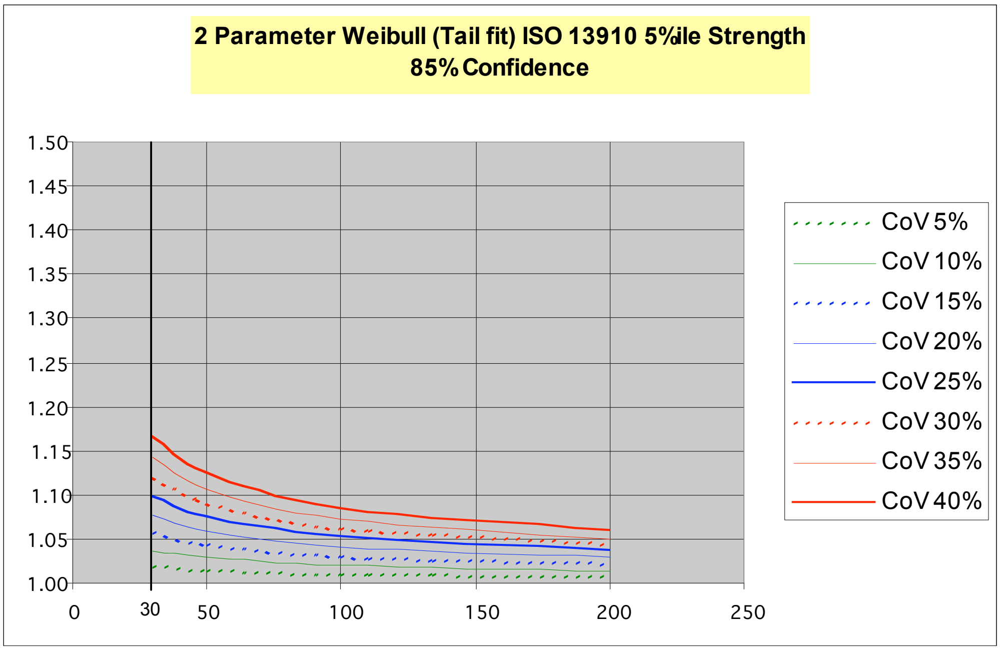
Equation
2 Parameter Weibull (Tail fit) ISO 13910
5%ile Strength

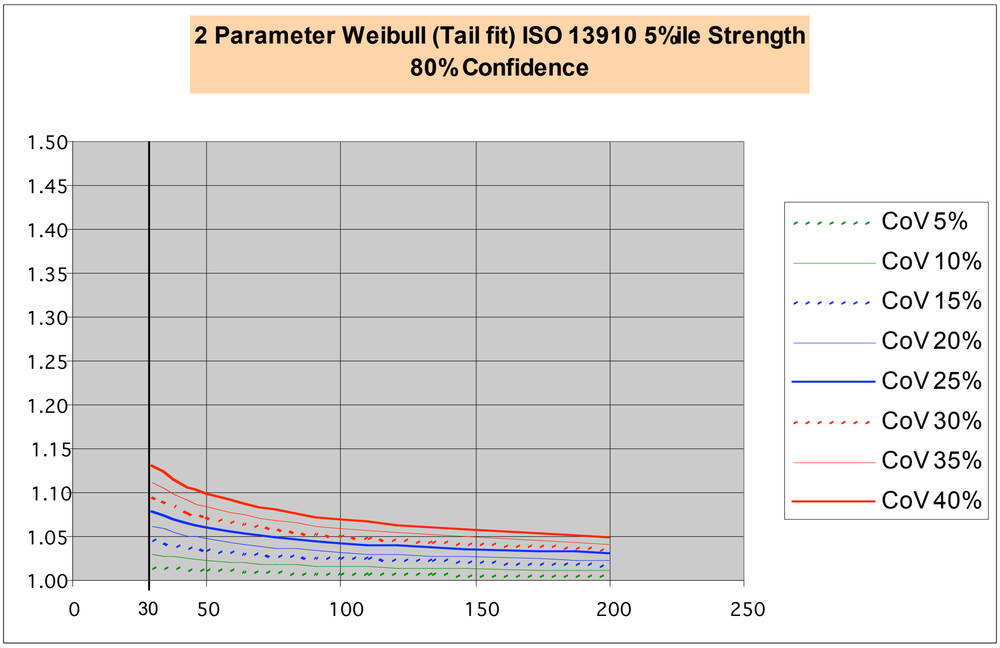
$$TCV = DV\left[\frac{1}{1 + A\dfrac{CoV}{\sqrt{n}}}\right]$$

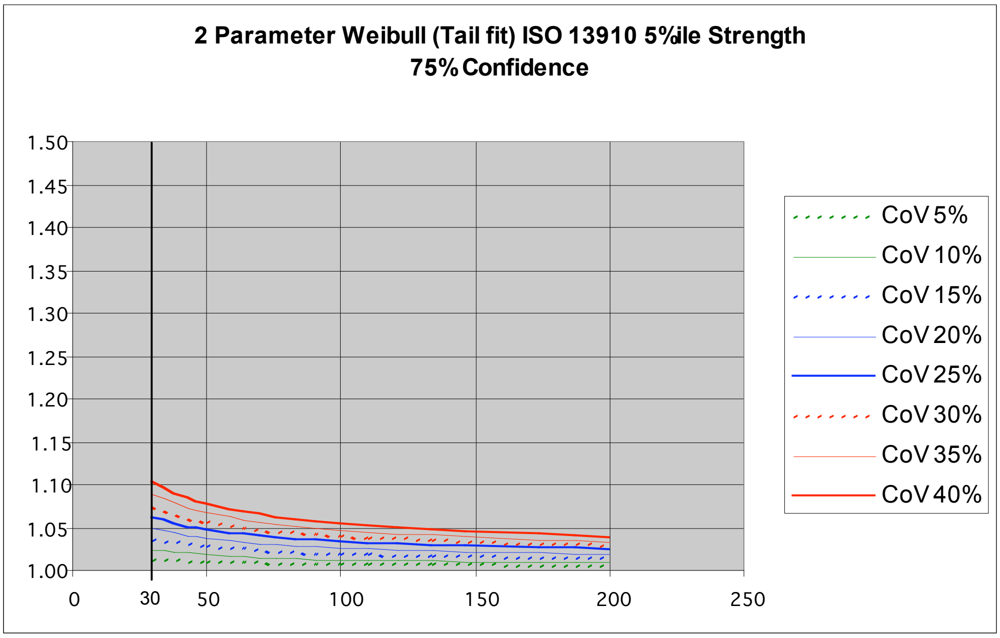
CL	*Value of A*
95%	-3.106
90%	-2.419
85%	-1.949
80%	-1.578
75%	-1.260


2 Parameter Weibull (Tail fit) ISO 13910 5%ile Strength
95% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


2 Parameter Weibull (Tail fit) ISO 13910 5%ile Strength
90% Confidence
Multiplier
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
30
50
100
150
200
Sample size
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


2 Parameter Weibull (Tail fit) ISO 13910 5%ile Strength
85% Confidence
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
30
50
100
150
200
250
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


2 Parameter Weibull (Tail fit) ISO 13910 5%ile Strength
80% Confidence
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
30
50
100
150
200
250
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%


2 Parameter Weibull (Tail fit) ISO 13910 5%ile Strength
75% Confidence
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0
30
50
100
150
200
250
CoV 5%
CoV 10%
CoV 15%
CoV 20%
CoV 25%
CoV 30%
CoV 35%
CoV 40%

Appendix C – Technical Support

C.1 Confidence Limits

C.1.1 Confidence in mean of Normal distribution

For normal distributions, the expression for a confidence limit on an estimation of the mean is given in (*eqn C.1*)[1,2].

$$CL(pr) = \mu + z(pr)\frac{\sigma}{\sqrt{n}} \qquad (eqn\ C.1)$$

with

CL(pr)	Confidence limit of estimate of mean
(pr)	Confidence level (eg 95%)
μ	Mean of sample (size *n*)
z(pr)	Normal distribution (a function of probability level)
σ	Standard deviation
N	Sample size (number of specimens in a single sample)
CoV	Coefficient of Variation = σ/μ

Then using $CoV = \dfrac{\sigma}{\mu}$ then $\sigma = \mu\ CoV$

And $CL(pr) = \mu + z(pr)\dfrac{\mu\ CoV}{\sqrt{n}}$

Hence $$CL(pr) = \mu\left(1 + z(pr)\frac{CoV}{\sqrt{n}}\right) \qquad (eqn\ C.2)$$

C.1.2 Confidence in any statistical distribution

(*eqn C.2*) can be generalised to other distributions (other than normal distribution) and other estimators (other than the mean – more importantly the 5%ile)[10].

$$CL(pr) = X\left(1 + A\frac{CoV}{\sqrt{n}}\right) \qquad (eqn\ C.3)$$

with

A	Constant for each Confidence level
X	Estimated property for the population

This is the origin of (*eqn C.24*) in this report. The values of *A* appropriate to each estimator[11] and method of analysis were determined using Monte Carlo simulations.

C.1.3 Design Values and Test Comparison Values

In AS/NZS 4063[10] and most other similar standards in other countries, a Design Value is found from a given confidence limit on a test value. Here we can make the substitution:

- *DV = CL(pr)* – the Design Value is the confidence limit on the test value *X*.

This then gives (*eqn C.3*) as

$$DV = X\left(1 + A\frac{CoV}{\sqrt{n}}\right)$$

and $$X_1 = DV\left[\frac{1}{\left(1 + A\frac{CoV}{\sqrt{n}}\right)}\right]$$

Here X_1 is the value of the test result that has the given confidence of being greater than the Design Value (DV). This is defined as the Test Comparison Value, giving

$$TCV = DV\left[\frac{1}{\left(1 + A\frac{CoV}{\sqrt{n}}\right)}\right] \qquad (eqn\ C.4)$$

with

TCV	Test Comparison Value (test value with given level of confidence that it meets the Design Value)
DV	Design Value
A	Constant for each Confidence level and each analysis method
n	Sample size (number of specimens in a single sample)
CoV	Coefficient of Variation = σ/μ

This is the origin of (*eqn 5.4*) and the multiplier in the square brackets in (*eqn C.4*) is the value *M* which is presented in all of the tables in Appendices A and B.

C.2 Methods of analysis

C.2.1 Non-parametric estimates

For non-parametric estimates from data, no attempt is made to fit a distribution through the data, it is simply treated as a list of numbers. It is outlined in Section 4.2.1

Non-parametric studies can make use of standard functions in msExcel:

- AVERAGE(multiple cell reference) returns the result of (*eqn C.5*)
- STDEV(multiple cell reference) returns the result of (*eqn C.7*)
- PERCENTILE(multiple cell reference, 0.05) gives an answer close to the value of the 5%ile calculated using the method outlined here.

The following nomenclature applies:

X_i	A test value. It is a typical value. The first value would be X_1, the second X_2, and a general one (the i^{th} one) X_i.
$\sum X_i$	The sum of all test values
$\overline{X}$	The arithmetic average of all test values
$X_{0.05}$	5^{th} percentile of the test values
s_x	The standard deviation of all test values
n	Sample size (number of specimens in a single sample)
CoV	Coefficient of Variation – see (*eqn C.6*)

Estimating the mean

The mean is the arithmetic average of all of the test data.

$$\overline{X} = \frac{\sum X_i}{n} \qquad \text{(eqn C.5)}$$

Estimating the Coefficient of Variation (CoV)

The Coefficient of Variation (CoV) is the ratio of standard deviation to the average.

$$CoV = \frac{s_x}{\overline{X}} \qquad \text{(eqn C.6)}$$

with $$s_x = \sqrt{\frac{\sum \left(X_i - \overline{X}\right)^2}{n}} \qquad \text{(eqn C.7)}$$

Estimating the 5%ile ($X_{0.05}$)

The non-parametric estimate of the 5%ile is simply an interpolation between the ranked data to give the value at the 5%ile level. Section 4.2.1 shows an example.

- All of the data is ranked (ordered from smallest to largest)
- The probability of each data point can be found from its ranking.
  - The first – lowest ranked – value, has $pr = \frac{1-0.5}{n}$
  - The second value, has $pr = \frac{2-0.5}{n}$

- The i^{th} value, has $pr = \frac{i - 0.5}{n}$

- The data point with the *pr* just greater than 0.05 is found ($X_{0.05+}$), and the one with the *pr* just less than 0.05 is found ($X_{0.05-}$).
- The value of $X_{0.05}$ is found by linear interpolation between $X_{0.05+}$ and $X_{0.05-}$.

C.2.2 Log-normal estimates

As outlined in Section 4.2.2, log-normal estimates can be found by fitting a log-normal distribution through the data, and finding critical parameters from the distribution that is fitted.

Fitting the distribution

The natural logarithm is found for all of the data.

$$v_i = \ln(X_i) \qquad (eqn\ C.8)$$

All of the subsequent operations are performed on the transformed data, using two parameters derived from the set of ln(*X*).

Mean of logs $m = \frac{\sum v_i}{n}$

Standard deviation of logs $s_v = \sqrt{\frac{\sum (v_i - m)^2}{n}}$

X_i	A test value. It is a typical value. The first value would be X_1, the second X_2, and a general one (the i^{th} one) X_i.
v_i	The natural log of the test values.
$\sum v_i$	The sum of all *v* values
m	The arithmetic average of all *v* values
$X_{0.05}$	5^{th} percentile of the log-normal distribution through the test values – see (*eqn C.11*)
s_v	The standard deviation of all *v* values
n	Sample size (number of specimens in a single sample)
CoV	Coefficient of Variation of the distribution through the data – see (*eqn C.10*)
exp()	The exponentiation function $\exp(s) = e^s$

Estimating the mean

The mean of the data is estimated from the distribution itself rather than the test values. It uses the values of *m* and s_v defined above:

$$\overline{X} = \exp\left(m + \frac{s_v^{\ 2}}{2}\right) \qquad (eqn\ C.9)$$

Estimating the Coefficient of Variation (CoV)
Again, the Coefficient of Variation (CoV) of the data can be estimated from the log-normal distribution fitted through the data rather than from the data itself. It uses the values of m and s_v defined above:

$$CoV = \sqrt{\exp\left(s_v^{\,2}\right) - 1} \qquad (eqn\ C.10)$$

Estimating the 5%ile ($X_{0.05}$)
The 5%ile of the data can be estimated from the log-normal distribution fitted through the data rather than from the data itself. It uses the values of m and s_v defined above:

$$X_{0.05} = \exp\left(m - 1.645 s_v\right) \qquad (eqn\ C.11)$$

C.2.3 Log-normal distribution estimates from tail data only

The fitting method outlined in Section C.2.2 required all of the test data, so could not be fitted to data from a proof testing program. However, the log-normal distribution can be fitted through just the tail data as shown in this section. Once the values of m and s_v have been found, then (*eqn C.9*), (*eqn C.10*) and (*eqn C.11*) can be used to find the estimates of mean, *CoV* and 5%ile of the data respectively.

Fitting the distribution

- The data is ranked from lowest to highest for which test information is available.
- The probability is assigned to each data point using the same relationships outlined in Section C.2.1. Here n is the total number of pieces where a test was started, not the number for which data was available. (eg if 30 pieces were loaded, but only 6 broke, then there is data for 6, so only 6 pieces can be ranked, but n = 30)
    - The first – lowest ranked – value, has $pr = \frac{1 - 0.5}{n}$
    - The second value, has $pr = \frac{2 - 0.5}{n}$
    - The i^{th} value, has $pr_i = \frac{i - 0.5}{n}$
- For each data point, $v_i = \ln(X_i)$ is plotted against $z(pr_i)$ = normal distribution points for each probability. The line of best fit is fitted through the plot of the data.
    - The value of m is given by the intercept on the v axis.
    - The value of s_v is given by the slope of the line of best fit.

C.2.4 Log-normal distribution from the mean of the data with specified CoV

In Section 4.2.3, it was noted that in fitting a distribution to the data, the character of the data may reflect sampling errors rather than the distribution of production properties from which the sample was drawn. It is possible to reflect the character of the population better by using the known CoV of the production property rather than the one suggested by the data from just a single sample.

In common with all log-normal distributions, the data is transformed to the natural logarithms using (*eqn C.8*).

$$v_i = \ln(X_i) \qquad \text{(copy } eqn\ C.8\text{)}$$

(*eqn C.10*) related *CoV* and the s_v value for the log-normal distribution. By inverting this expression, the known population value of CoV can give the required s_v directly.

$$CoV = \sqrt{\exp(s_v^{\,2}) - 1} \qquad \text{(copy } eqn\ C.10\text{)}$$

$$CoV^2 = \exp(s_v^{\,2}) - 1$$

$$\exp(s_v^{\,2}) = CoV^2 + 1$$

$$s_v^{\,2} = \ln(CoV^2 + 1)$$

$$s_v = \sqrt{\ln(CoV^2 + 1)} \qquad (eqn\ C.12)$$

X_i	A test value. It is a typical value. The first value would be X_1, the second X_2, and a general one (the i^{th} one) X_i.
v_i	The natural log of the test values.
$\sum v_i$	The sum of all *v* values
m	The arithmetic average of all *v* values
$X_{0.05}$	5th percentile of the log-normal distribution through the test values – see (*eqn C.11*)
s_v	The standard deviation of all *v* values
n	Sample size (number of specimens in a single sample)
CoV	Coefficient of Variation of the distribution through the data – see (*eqn C.10*)
z(*pr*)	Points on the normal distribution – function of probability
exp()	The exponentiation function eg. exp(*s*) = e^s

s_v is calculated using only the *CoV* that is expected for the grade of timber that has been sampled. The value of *m* can be found from the test data and the value of s_v found from (*eqn C.12*).

- The data is ranked from lowest to highest.
- The probability is assigned to each data point using the same relationships outlined in Section C.2.1.
    - The first – lowest ranked – value, has $pr = \frac{1 - 0.5}{n}$
    - The second value, has $pr = \frac{2 - 0.5}{n}$
    - The i^{th} value, has $pr_i = \frac{i - 0.5}{n}$
- *m* is evaluated using (*eqn C.13*)

$$m = \frac{\sum v_i}{n} - s_v \frac{\sum z(pr_i)}{n} \qquad (eqn\ C.13)$$

with $z(pr_i)$ the normal distribution points for probability of each point (pr_i)

Once the values of m and s_v have been found, then (*eqn C.9*), (*eqn C.10*) and (*eqn C.11*) can be used to find the estimates of mean, *CoV* and 5%ile of the data respectively.

In using this method of analysis, a single value of CoV is used, but because, there will be a small variation in CoV of the production from time to time, the actual value of the CoV of production may differ a little from the value used in the analysis. In modelling the analysis method, allowance is made for the variation in CoV (see Section C.3.3).

C.2.5 Log-normal distribution from the mean of the data with specified CoV using tail fit only

With this method of evaluation of the critical properties of the data, a log-normal distribution is fitted through the data points available, but the distribution is forced to have the CoV known for the grade produced.

The method is similar to the one presented in Section C.2.4. In common with all log-normal distributions, the data is transformed to the natural logarithms using (*eqnC.8*).

$$v_i = \ln(X_i) \qquad \text{(copy } eqn\ C.8\text{)}$$

s_v is calculated using only the *CoV* that is expected for the grade of timber that has been sampled using (*eqn C.12*).

$$s_v = \sqrt{\ln\left(CoV^2 + 1\right)} \qquad \text{(copy } eqn\ C.12\text{)}$$

The value of m can be found from the test data and the value of s_v found from (*eqn C.12*).

- The data is ranked from lowest to highest for which test information is available. Here n is the total number of pieces where a test was started, not the number for which data was available. (eg if 30 pieces were loaded, but only 6 broke, then there is data for 6, so only 6 pieces can be ranked, but $n = 30$)
- The probability is assigned to each data point using the same relationships outlined in Section C.2.1.
  - The first – lowest ranked – value, has $pr = \frac{1 - 0.5}{n}$
  - The second value, has $pr = \frac{2 - 0.5}{n}$
  - The i^{th} value, has $pr_i = \frac{i - 0.5}{n}$
- m is evaluated using (*eqn C.14*) but in this case, the divisor is n_b – the number of pieces that broke and for which strength information is available.

$$m = \frac{\sum v_i}{n_b} - s_v \frac{\sum z(pr_i)}{n_b} \qquad (eqn\ C.14)$$

with $z(pr_i)$ the normal distribution points for probability of each point (pr_i)

X_i	A test value. It is a typical value. The first value would be X_1, the second X_2, and a general one (the i^{th} one) X_i.
v_i	The natural log of the test values.
$\sum v_i$	The sum of all *v* values
m	The arithmetic average of all *v* values
$X_{0.05}$	5^{th} percentile of the log-normal distribution through the test values – see (*eqn C.11*)
s_v	The standard deviation of all *v* values
n	Sample size (number of specimens in a single sample)
n_b	Number of pieces in the sample that broke and for which strength data is available.
CoV	Coefficient of Variation of the distribution through the data – see (*eqn C.10*)
z(*pr*)	Points on the normal distribution – function of probability
exp()	The exponentiation function eg. $\exp(s) = e^s$

Once the values of *m* and s_v have been found, then (*eqn C.9*), (*eqn C.10*) and (*eqn C.11*) can be used to find the estimates of mean, *CoV* and 5%ile of the data respectively.

In using this method of analysis, a single value of CoV is used, but because, there will be a small variation in CoV of the production from time to time, the actual value of the CoV of production may differ a little from the value used in the analysis. In modelling the analysis method, allowance is made for the variation in CoV (see Section C.3.3).

C.2.6 Two parameter Weibull estimates

Tail fitting a 2 parameter Weibull (2pW) distribution is used in US standards and has been adopted in a number of mills for monitoring programs. Under the US standard, the tail is defined as the larger of (the lowest 15% or the lowest 15 pieces).

The tail fit 2 parameter Weibull distribution appears to have a good match for the tail data, but gives a *CoV* that is significantly lower than the *CoV* of the whole distribution. For sample sizes of less than 100, the definition of the tail gives 15 pieces as the limiting criteria, and that will represent something more than 15% of the data. This will change the ratio of the 2pW CoV to the whole of data CoV. The errors in this slight discrepancy were not significant for sample sizes of 30 and of 50, but they were for sample sizes less than 30. For this reason, the study restricted the use of tail-fit data for the 2pW distribution to samples of 30 or more pieces.

The 2 parameter Weibull distribution is characterised by two parameters that have a variety of symbols. In this report, the symbols *a* and *b* will be used to define the distribution.

- The shape parameter = 1/*a*
- The scale parameter = exp(*b*)

Once these parameters have been evaluated, then the mean, CoV and 5%ile can be estimated from the distribution using *a* and *b*[16].

- The data is ranked from lowest to highest for which test information is available. Here *n* is the total number of pieces where a test was started, not the number for which data was available. (eg if 150 pieces were loaded, but only 23 are used as the tail, then only the lowest 23 pieces are used, but *n* = 150)
- The probability is assigned to each data point using the same relationships outlined in Section C.2.1.
    - The first – lowest ranked – value, has $pr = \frac{1-0.5}{n}$
    - The second value, has $pr = \frac{2-0.5}{n}$
    - The i^{th} value, has $pr_i = \frac{i-0.5}{n}$
- For each point the natural logarithm of the strength is used (*eqn C.8*) and this is plotted against ln(-ln(1-pr_i)).
    - The slope of this plot gives *a*.
    - The intercept on the *v* axis gives *b*.

v_i	The natural log of the test values.
a	A parameter of 2 parameter Weibull distribution shape factor = 1/*a*
b	A parameter of 2 parameter Weibull distribution scale factor = exp(*b*)
$X_{0.05}$	5th percentile of the log-normal distribution through the test values – see (*eqn C.17*)
n	Sample size (number of specimens in a single sample)
CoV	Coefficient of Variation of the distribution through the data – see (*eqn C.16*)
Γ()	Gamma function
exp()	The exponentiation function eg. exp(*s*) = e^s

The calculation of the mean and standard deviation of the distribution both make use of the gamma function Γ

Mean $\overline{X} = \exp(b)\Gamma(a+1)$ (*eqn C.15*)

This can be found in msExcel by EXP(b)*EXP(GAMMALN(a+1))

CoV $CoV = \frac{\sqrt{\Gamma(2a+1) - \Gamma(a+1)^2}}{\Gamma(a+1)}$ (*eqn C.16*)

In msExcel (EXP(GAMMALN(2a+1))-EXP(GAMMALN((a+1)^2))^0.5/ (EXP(GAMMALN(a+1)))

5%ile $X_{0.05} = \exp(b) \times (-\ln(1 - pr_i))^a$ (*eqn C.17*)

ISO 13910 includes an example of a 2 parameter Weibull fit to tail data for the estimation of 5%ile strength. It is very similar to the above example, except that it discards the lowest two values.

- The data is ranked from lowest to highest for which test information is available. Here *n* is the total number of pieces where a test was started, not the number for which data was available. (eg if 150 pieces were loaded, but only 23 are used as the tail, then only the lowest 23 pieces are used, but *n* = 150)
- The probability is assigned to each data point using the same relationships outlined in Section C.2.1.
    - The first – lowest ranked – value, has $pr = \frac{1 - 0.5}{n}$
    - The second value, has $pr = \frac{2 - 0.5}{n}$
    - The i^{th} value, has $pr_i = \frac{i - 0.5}{n}$
- The data for the lowest two points are then ignored so the data starts with the point $pr = \frac{3 - 0.5}{n} = \frac{2.5}{n}$
- For each point the natural logarithm of the strength is used (*eqn C.8*) and this is plotted against ln(-ln(1-pr_i)).
    - The slope of this plot gives *a*.
    - The intercept on the *v* axis gives *b*.

The analysis proceeds as indicated above, but the removal of the lowest two points seems to give this method more stability.

C.3 Monte Carlo Simulations

The process of Monte Carlo Simulation is one of repeating a number of analyses of data drawn at random from distributions of "raw data". (Its name is derived from the code name of similar studies performed in World War II.)

C.3.1 Monte Carlo simulations for this study

In the context of this study, whole production runs of timber were modelled using a statistical distribution. Section 4.2 discusses the process of fitting distributions to data sets of MoE, bending strength and tension strength. Monte Carlo simulations were performed using @RISK software[3] – an msExcel add in by Palisade Corp.

- The distributions with appropriate properties (*CoV*, 5%ile for strength and mean for MoE) were defined in a spreadsheet. As the distribution defined all of the timber in the production run, the characteristics of the raw data were the critical points of the input distribution.
  - The average of the production run was the average of the input distribution,
  - The CoV of the production run was the CoV of the input distribution,
  - The 5%ile was the 5%ile of the input distribution.
- For each simulation run, a number of pieces per sample was defined (eg 5, 10, 20, …) and values were selected at random from the distributions of the production run for each piece in the sample.
- Each sample was operated on to estimate the critical points for the production (mean, CoV, 5%ile). The methods used to estimate these were described in Section C.2. For each simulation run, the properties were estimated for all of the analysis methods from the same sample, giving a direct comparison between all of the analysis methods.
- Simulation runs were repeated.
  - For strength, 2500 simulation runs were used for each sample size and CoV combination.
  - For MoE, 1000 simulation runs were used for each sample size and CoV combination.
- Each simulation gave a distribution for each estimate, so the results could be interpreted directly as confidence limits.
  - Table C.1 gives an example of the output for a simulation of MoE.
  - Table C.2 gives an example of the output for a simulation of strength.

Table C.1 Simulation Output for MoE (production CoV 15%)

Production Data	4
CoV of population	0.15
no of specimens	30
pop 5%ile	8.124573
Mean	10.5
std deviation	1.575

	average nonparm	**Average ln**	**CoV full nonparm**	**CoV full ln**	**5%ile full nonparm**	**5%ile full ln**
minimum	9.566862	9.57172	0.100254	9.81E-02	6.87E+00	7.035088
maximum	11.42296	11.41863	0.231226	0.212373	9.808073	9.153103
mean	10.50024	10.5041	0.14845	0.148643	8.311314	8.150551
st deviation	0.295478	2.96E-01	1.99E-02	1.92E-02	4.12E-01	0.347304
errors	0	0	0	0	0	0
5%	10.00369	10.01177	0.117385	0.118519	7.602729	7.559521
10%	10.12494	10.1305	0.124278	0.124936	7.781636	7.688518
15%	10.19867	10.20321	0.1284	0.129082	7.895227	7.797836
20%	10.24428	10.24816	0.131826	0.132309	7.977114	7.859321
25%	10.29872	10.30053	0.134784	0.135088	8.036125	7.909787
30%	10.34139	10.34396	0.137535	0.137964	8.090206	7.963676
35%	10.3784	10.38211	0.140099	0.141036	8.166368	8.023146
40%	10.41615	10.41885	0.142563	0.143446	8.216382	8.063827
45%	10.4585	10.46219	0.144761	0.145318	8.272581	8.114374
50%	10.49579	10.49927	0.146606	0.147627	8.323778	8.150947
55%	10.53401	10.53872	0.149173	0.150247	8.380261	8.192926
60%	10.57351	10.57658	0.152212	0.152448	8.42844	8.238899
65%	10.61409	10.61863	0.154719	0.154686	8.47611	8.287342
70%	10.65934	10.66178	0.157611	0.157534	8.529819	8.345545
75%	10.69678	10.70255	0.160811	0.160221	8.585226	8.399123
80%	10.7553	10.75742	0.163729	0.163868	8.66181	8.45155
85%	10.8125	10.81488	0.168346	0.168718	8.731902	8.512689
90%	10.8708	10.8732	0.174406	0.173815	8.843313	8.590223
95%	11.00238	11.00884	0.184841	0.182686	8.974718	8.702073

The data in the 5% row gives the 5%ile probability of the estimate of the mean MoE or 5%ile MoE.
Likewise the data in the 10% row gives the 10%ile probability, the data in the 15% row gives the 15% probability, etc.

The data in the row headed "mean" gives the average of all of the estimations. Where it is higher than the true average of the production (in this case, 10.5 GPa), there is a systematic over-estimation of the property.

For the strength evaluation, the table was a little different because some of the analysis methods used proof testing and for those methods, the proof stress was also an input, as it established the tail of the distribution. The proof stress is also passed to the analysis as shown in Table C.2.

Table C.2 Simulation Output for strength (production CoV 35%)

Production Data	10
CoV of population	0.35
CoV of tail	0.171939
no of specimens	20
proof stress	25
pop 5%ile	20
Mean	37.06454
std deviation	12.97259
iterations	2500

	count tail	5%ile full nonparm	5%ile full ln	5%ile tail ln best fit	5%ile tail ln CoV	5%ile ln fr mean CoV	5%ile tail 2pW best fit
minimum	0	12.0123	13.18462	12.23558	13.10572	15.37462	12.65374
maximum	10	31.51909	28.99482	24.79607	27.78092	26.48254	24.80392
mean	3.2292	21.71755	20.28748	19.90126	20.10625	20.05742	20.01205
st deviation	1.614221	2.82568	2.375791	2.24854	2.585688	1.521345	2.210343
errors	0	0	0	336	68	0	336
5%	1	17.21329	16.56301	16.0288	16.5432	17.65149	16.22705
10%	1	18.1235	17.28318	16.92358	17.09794	18.16251	17.07866
15%	2	18.7527	17.81447	17.4691	17.54026	18.49562	17.62907
20%	2	19.29856	18.25469	17.93437	17.90624	18.75236	18.05557
25%	2	19.82951	18.6263	18.36271	18.22482	19.01094	18.50735
30%	2	20.22642	18.9627	18.72441	18.5883	19.23816	18.85052
35%	2	20.6048	19.30079	19.04301	18.92614	19.4678	19.17792
40%	3	20.9276	19.62585	19.36721	19.19468	19.63463	19.50288
45%	3	21.307	19.90398	19.6982	19.50868	19.82478	19.81574
50%	3	21.64615	20.1936	20.00205	19.75258	19.99656	20.11393
55%	3	21.97873	20.47451	20.2742	20.01671	20.17908	20.39157
60%	4	22.3427	20.77836	20.60618	20.3201	20.3554	20.70707
65%	4	22.71919	21.14427	20.88702	20.68761	20.59343	20.99132
70%	4	23.16552	21.4905	21.19899	21.09094	20.79559	21.2913
75%	4	23.55849	21.81982	21.5602	21.5595	21.03709	21.64373
80%	5	24.09635	22.23398	21.89019	22.1047	21.29689	21.98604
85%	5	24.61761	22.6906	22.32393	22.81242	21.62568	22.40273
90%	5	25.33358	23.38523	22.81669	23.79504	22.05161	22.88081
95%	6	26.62834	24.3536	23.51835	25.08858	22.66244	23.55066

The data in the 5% row gives the 5% probability estimate of the 5%ile strength. Likewise the data in the 10% row gives the 10% probability estimate, the data in the 15% row gives the 15% probability estimate, etc.

The data in the row headed "mean" gives the average of all of the estimations. Where it is higher than the true 5%ile of the production (in this case, 20 MPa), there is a systematic over-estimation of the property.

C.3.2 Determination of Confidence limits for analysis methods

The data in Tables C.1 and C.2 represented a single combination of production CoV and sample size. In the case of the strength data shown in Table C.2, it also represented a single value of proof stress.

All of the data for a single analysis method was combined to establish values for *A* in (*eqn 5.3*) or (*eqn 5.4*). It is the same value of *A*.

Confidence level from the data
Because some analysis methods systematically over-estimated or under-estimated the production value (refer Section C.4.1), and because of slight skewness in some of the distributions of estimates, it was not straight forward to estimate the sampling error.

- Sampling errors are found for different confidence levels. (For example at the 95% confidence level, the basis for the calculations will be the 5%ile or 95%ile level of the estimations.)
- Sampling error magnitude was the difference between the percentile point of the estimation and the mean estimation of the same property.
- Over-estimation was the difference between the mean estimation and the production value of the same property.

The relationships are depicted in Figure C.1.

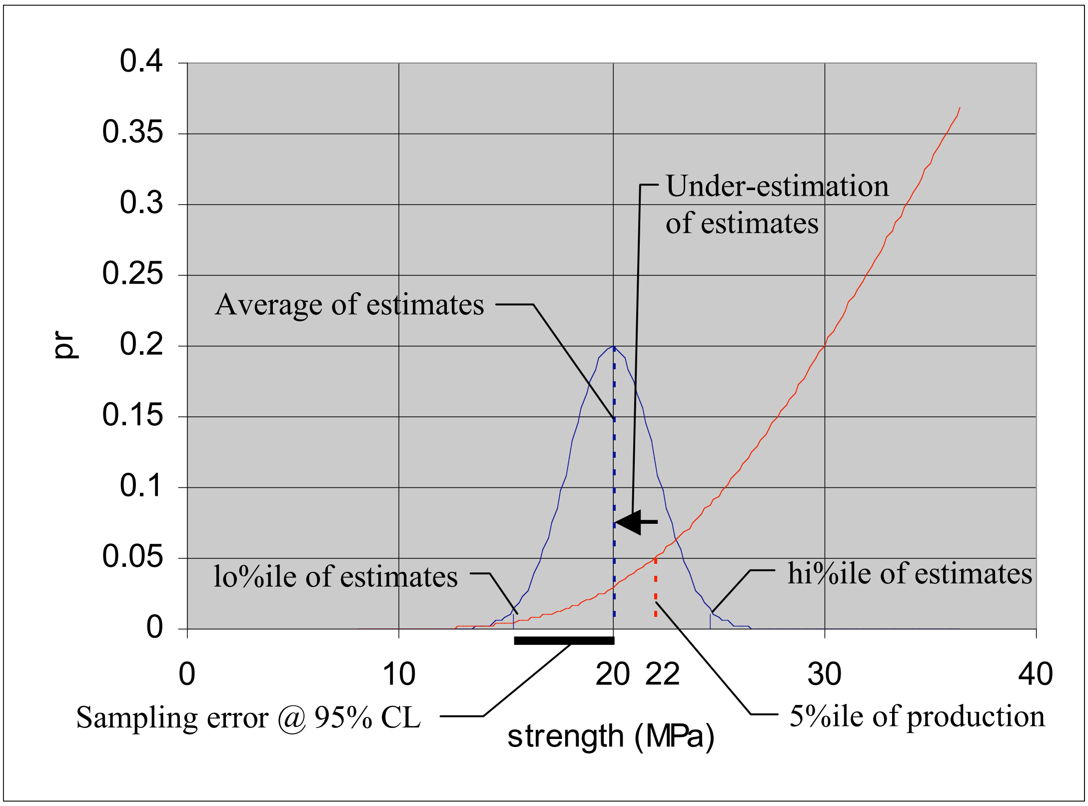


Figure C.1 – Representation of Estimate of production property

Figure C.1 shows the distribution of estimations of the 5%ile strength of a production. The production distribution is shown in red and the distribution of the estimate is shown in blue.

- The distribution estimate is close to the tail of the production distribution.
- In Figure C.1 there is a slight tendency to under-estimate the 5%ile. (The average estimate is 20 MPa while the true value was 22 MPa.)

- At the 95%ile level, the sampling error will be given by the 5%ile of the estimates and the 95%ile of the estimates.

For this study, the sampling error at a given confidence level (*CL)* was given by:
(*upper CL value – lower CL value*)/2 (*eqn C.18*)

For this study, the over-estimation of the estimate was given by:
(*average estimation – true production value*) (*eqn C.19*)

- Where there is a systematic over- estimation this will be a positive value.
- Where there is a systematic under-estimation (as illustrated in Figure C.1), then this will be a negative value.
- This is independent of confidence level.

The lower bound on the estimate for a given Confidence Level is given by *L*(*CL*) as shown in (*eqn C.20*):

L(*CL*) = *Test value – over-estimation – sampling error* (*CL*) (*eqn C.20*)

The over-estimation is independent of Confidence Level, and only a function of the method of analysis. However, the sampling error is both a function of the method of analysis and the Confidence Level.

In general, unless the method significantly under-estimates the property, *L*(*CL*) will be less than the true value of the parent distribution.

This can be used in (*eqn 5.2*) to evaluate the value of *A*.

$$L = X\left(1 + A\frac{CoV}{\sqrt{n}}\right) \qquad \text{(copy } eqn\ 5.2)$$

Hence $$\frac{L}{X} = \left(1 + A\frac{CoV}{\sqrt{n}}\right) \qquad (eqn\ C.21)$$

This ratio was used to give a non-dimensional value which could be used to find A directly for a given combination of analysis method and Confidence Level.

For a given analysis method and Confidence Level, there are a number of results for different CoV and sample size combinations.

The values of $\frac{L}{X}$ could be plotted against $\frac{CoV}{\sqrt{n}}$ and the slope of this line gives *A*.

Also, to confirm that the power of *n* in (*eqn C.21*) should be 0.5, a more general equation was solved. (*Eqn C.21*) can be rewritten with a general exponent of *n* and then the optimum value of this can be found from analysis.

(*eqn C.21*) $$\frac{L}{X} = \left(1 + A\frac{CoV}{(n)^{\beta}}\right)$$

hence $$\frac{L}{X} - 1 = A\frac{CoV}{(n)^{\beta}}$$

and $$\ln\left(\frac{L}{X}-1\right) = \ln(A) + \ln(CoV) - B\ln(n)$$

$$\ln\left(\frac{\frac{L}{X}-1}{CoV}\right) = \ln(A) - B\ln(n) \qquad (eqn\ C.22)$$

In this case by plotting $\ln\left(\frac{\frac{L}{X}-1}{CoV}\right)$ against $\ln(n)$, then the slope gives –*B* and the intercept ln(*A*). In this way the optimum index for *n* can be evaluated for each analysis method. The results invariably gave answers near 0.5, confirming the form of the (*eqn 5.2*)

C.3.3 Variation in CoV used in prescribed CoV analyses

For the analysis methods in which a prescribed CoV is used, the value chosen by a producer may not match the value of the production. The use of a CoV that is slightly different to the true value of the material will introduce further errors in the analysis.

It is possible to determine the effect of variability in the prescribed CoV value, and this was tested with a Monte Carlo simulation. Two levels of variability in CoV were chosen as described in Section 4.1.3. The analyses gave repeatable results.

C.3.4 Success of data simulation

A major key to the success of Monte Carlo simulation is to create distributions that correctly model the data.[8] A number of different data sets were used to fit distributions for each of the modelled parameters.

A distribution is used to model a single property for a single grade in a single production run. It simulates the testing of every piece of timber in-grade in the run. There are no data sets to compare this with, but a number of other data sets were used in the study.

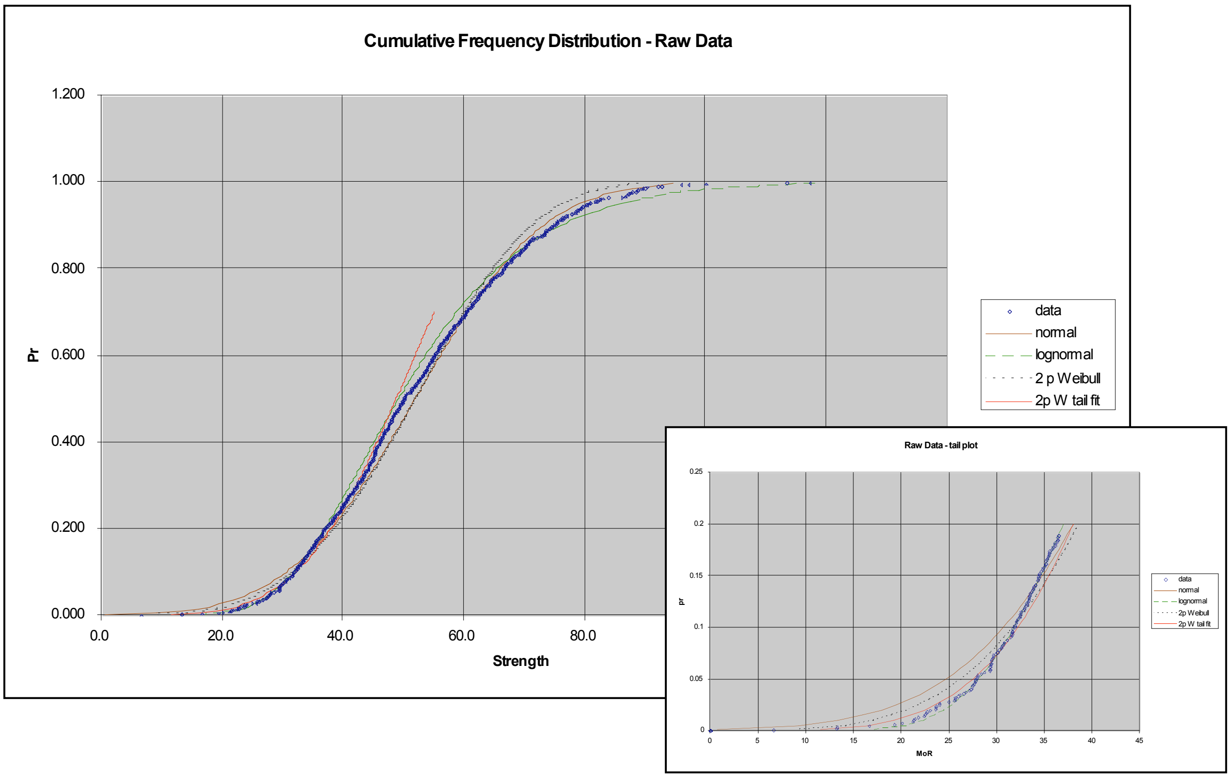


Figure C.2 – Data fit to bending strength data

C.3.4.1 Bending strength data

Figure C.2 shows some test bending strength data for a single mill and a single grade with a number of distributions fitted to it.

- The log-normal distribution fitted the data well over the complete range of the data.
- In the region of the lower tail, the log-normal distribution also followed the data well.

The log-normal distribution was used to model the strength of production. In determining the sensitivity of the analysis method to the assumption of the distribution of the production properties, the following distributions were also tried:

- Beta general
- Gamma
- Inverse Gaussian

The results of the simulations did not differ with the different distributions tried. It was decided to use the log-normal distribution because it was understood already in the industry and it fitted the bending strength data as well as any of the others tried.

C.3.4.2 MoE data

Figure C.3 shows some MoE data for a single mill and a single grade with a number of distributions fitted to the data.

The MoE data fitted quite well to the log-normal distribution, and while the normal distribution give an accurate prediction of the average value and was well aligned

with the data near the top of the distribution, there was a misalignment between the normal distribution and the data at the tail.
The log-normal distribution was used to model the stiffness data from production runs.

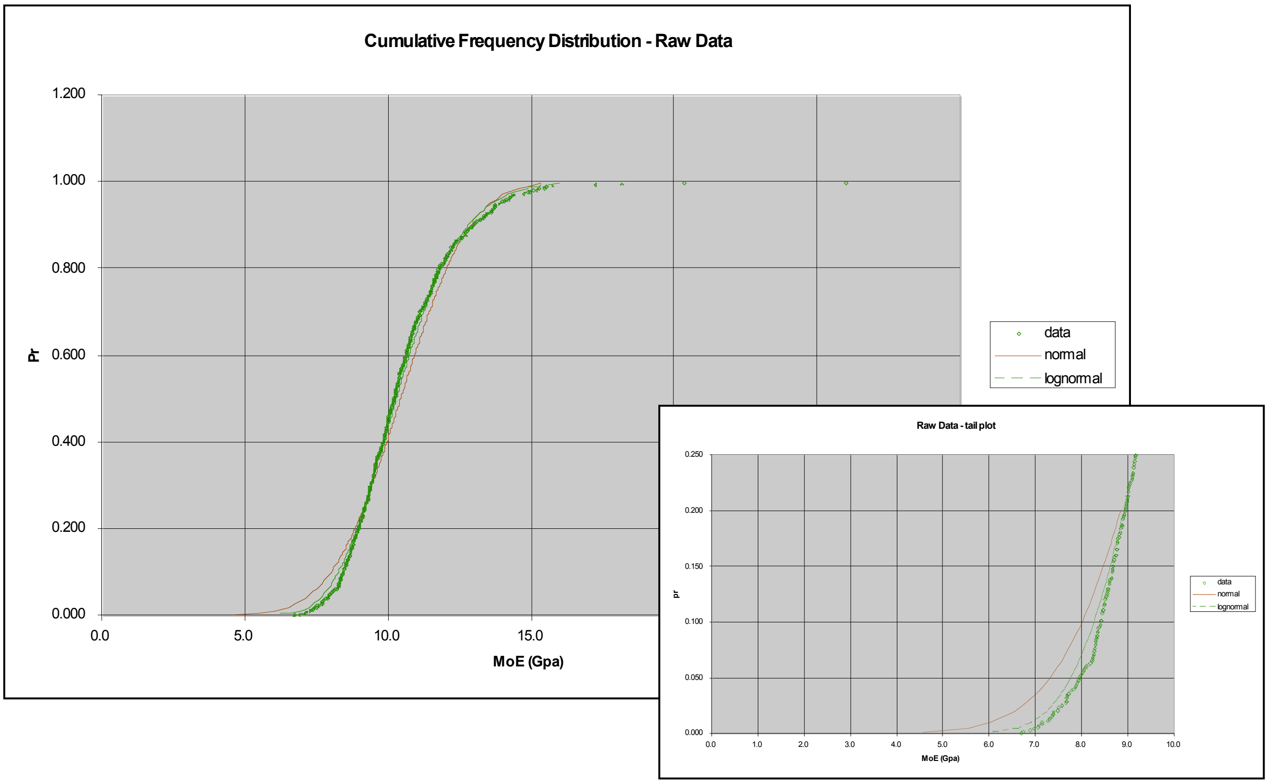


Figure C.3 – Data fit to MoE data

C.3.4.3 Tension strength data

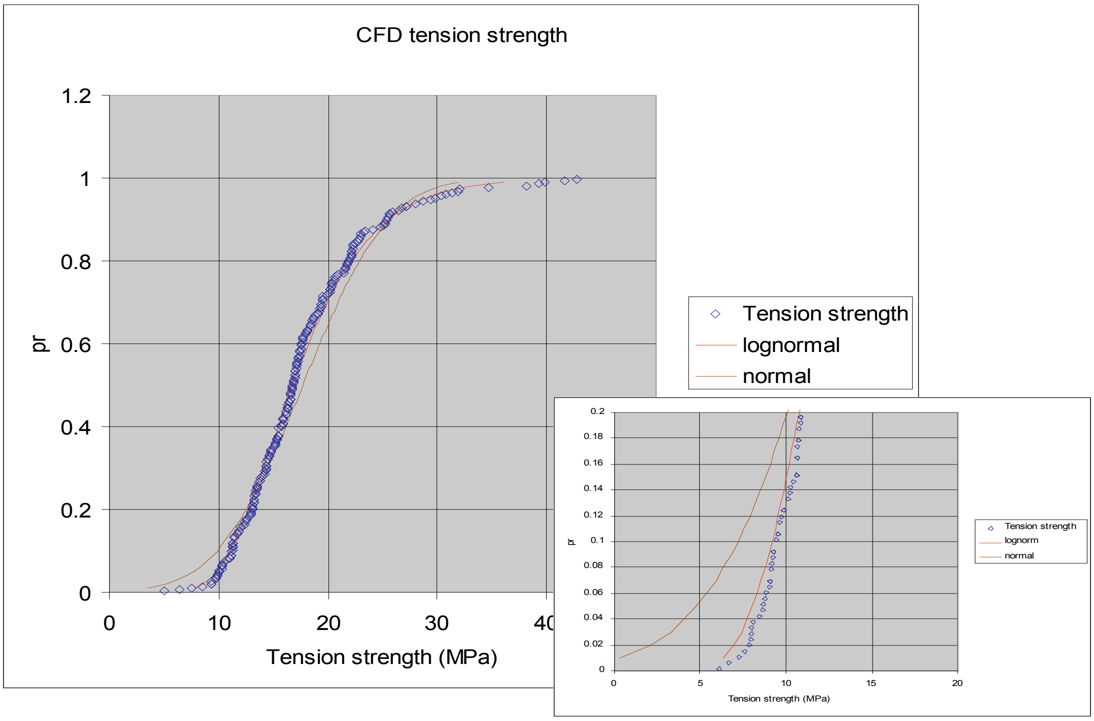


Figure C.4 – Data fit to tension strength data

Again, the tension strength data was well modelled by the log-normal distribution.

C.4 Results

C.4.1 Under and over-estimation of properties

Different methods of analysis only produce estimates of the property for the production over a given period, and can systematically underestimate or over-estimate the estimated property.
In general:

- In estimating the mean of a property, there is little systematic under-estimation or over-estimation.
- In estimating the 5%ile of a property, methods that use all test data overestimate the 5%ile.
- For larger numbers of specimens in a sample, there is lower magnitude of under- or over-estimation.
- For higher CoV of the property being measured, there is a higher magnitude of under- or over-estimation.

In this study, the following measure of over-estimation was used:

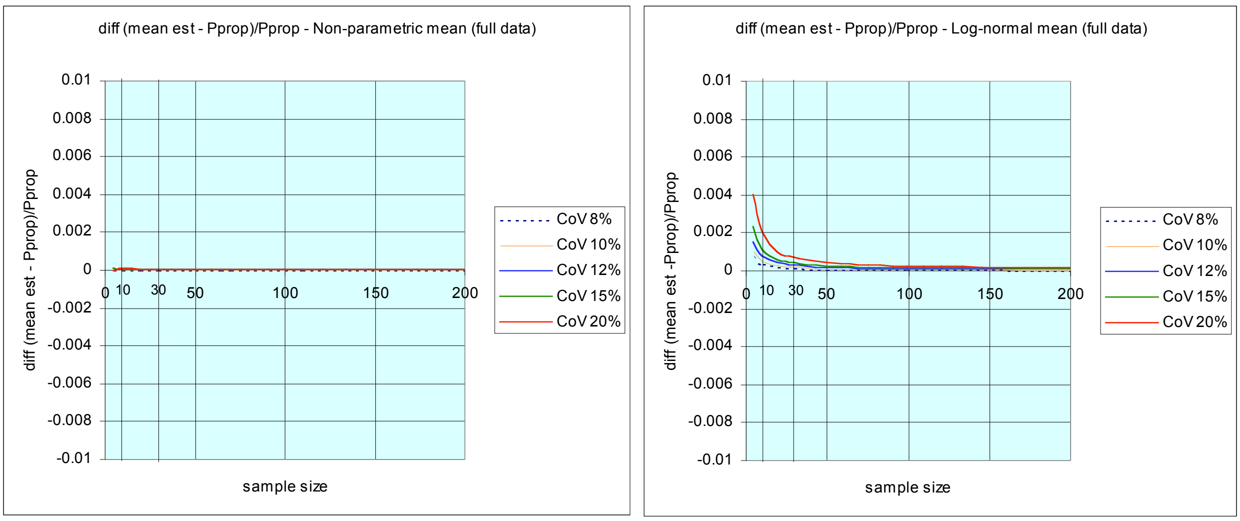
$$\text{Over-estimation} = \frac{(average\ value - production\ property)}{production\ property} \qquad \textit{(eqn C.23)}$$

This gives an estimate of the systematic over-estimation of the property as a fraction of the true value of the property. Note that this is not necessarily the situation in every case, as can be seen in the illustration in Table C.2. However, over a large number of estimations, a general trend can be seen for a given analysis method to under- or over-estimate the production parameter.

To facilitate comparison, all of the plots within a single Figure will have the same scale.

C.4.1.1 Estimation of the mean and 5%ile MoE

For modelling MoE, 1000 simulations were used. Statistical methods are quite robust in estimating the mean of a sample, and this was confirmed in the study.



(a) non-parametric estimation of mean (b) estimation of mean using log-normal

Figure C.5 – Over-estimation of mean MoE

Figure C.5 shows that:

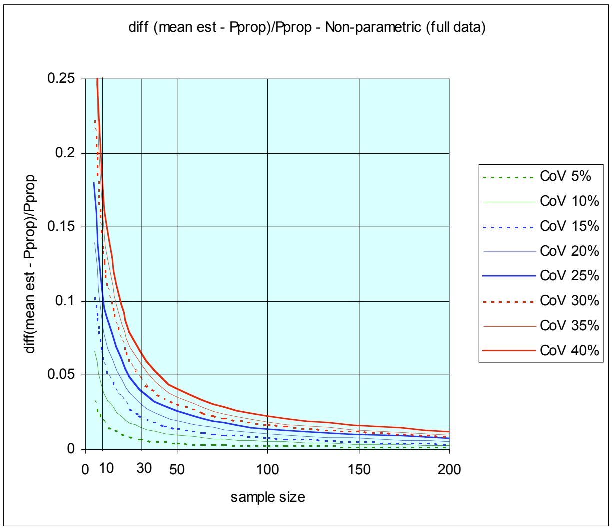
- There was no systematic error for any of the evaluations of mean MoE using the arithmetic mean of the test samples. (Figure C.5(a))
- However, where a log-normal distribution was fitted through the data, and the mean found from the log-normal distribution, there was a very small tendency to over-estimate the mean MoE from the data. Figure C.5(b) shows that this was generally less than 0.1% of the true mean MoE for 20 or more specimens per sample.

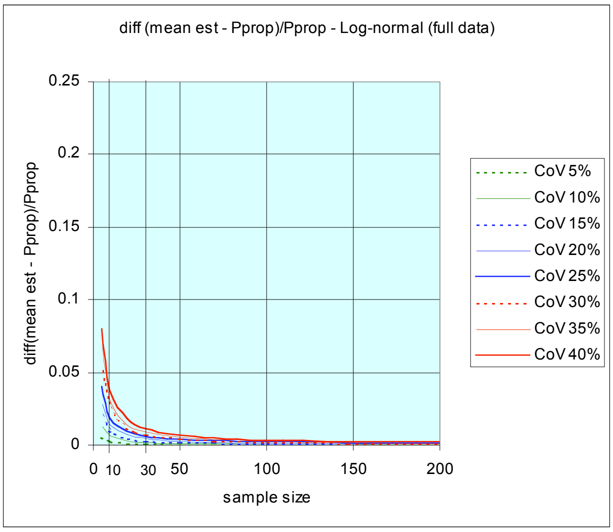
The systematic overestimation of the mean MoE by either analysis was negligible.

In contrast to estimations of the mean MoE, the study showed that both methods gave over-estimation of the 5%ile MoE. Figure C.6 shows that:

- For evaluations of 5%ile MoE using non-parametric methods on the test data, all data sets show an over-estimation of the 5%ile MoE. (Figure C.6(a)). The over-estimation is highest for small sample sizes and high CoV products. For sample sizes of 20 or more, normal CoV for MGP timber gives a systematic over-estimation of 5%ile MoE of 3% or less.
- Where a log-normal distribution was fitted through the data, and the 5%ile found from the fitted log-normal distribution, there was a much smaller tendency to over-estimate the 5%ile MoE from the data. Figure C.6(b) shows that this was generally less than 0.4% of the true 5%ile MoE for 20 or more specimens per sample. This is around 1/10th the systematic error of non-parametric estimations of 5%ile MoE

The systematic overestimation of the 5%ile MoE by either analysis was small, but much smaller for the 5%ile estimated from a log-normal distribution fitted through the data. Any tendency to over- or under-estimate production properties has been taken into account in the findings of the study.





(a) non-parametric estimation of 5%ile (b) estimation of 5%ile using log-normal

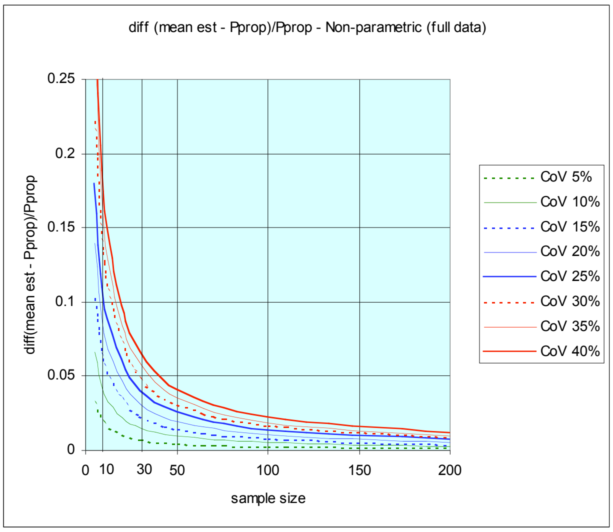
Figure C.6 – Over-estimation of 5%ile MoE

C.4.1.2 Estimation of 5%ile Strength from all test data

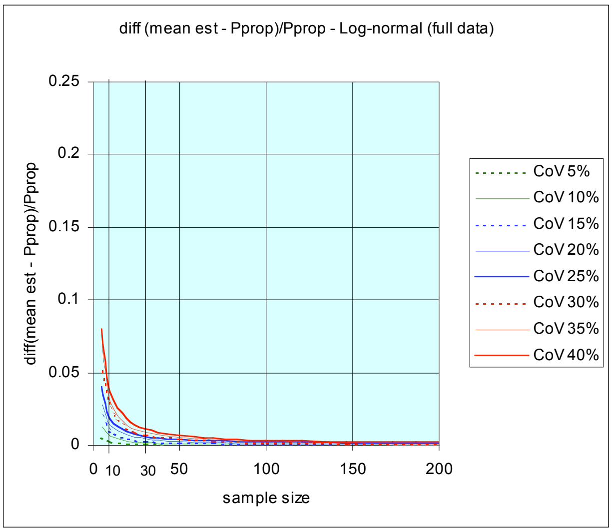
The key parameter for estimation of strength is the 5%ile strength. As there was more variability in strength data compared with MoE data, 2500 simulations were used to determine the distribution of the estimation of strength.

Figure C.7 shows the systematic overestimation of the 5%ile strength from analysis methods that use all of the data from the test series. (All test specimens are loaded to failure, and all failure loads are used in the analysis.) Figure C.7 shows that:

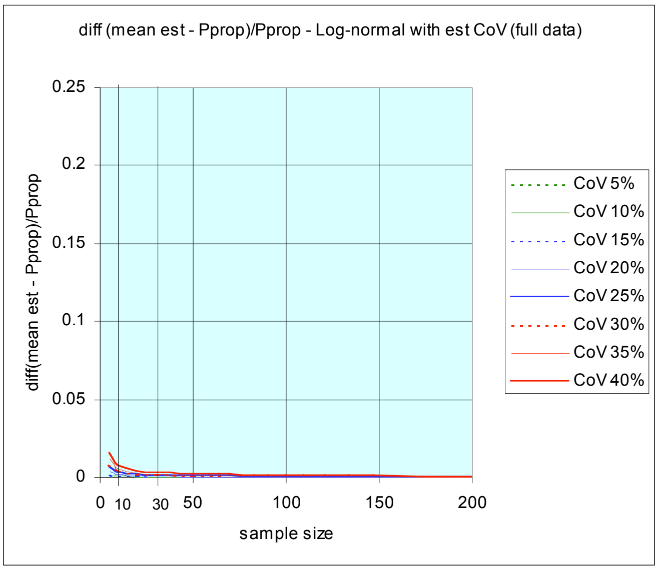
- The 5%ile is generally over-estimated by all analysis methods that use all of the test data.
- The systematic over-estimation of the non-parametric methods was significantly greater than for the methods based on a log-normal fit through the data.
- The overestimation is greater for higher coefficients of variation and for smaller sample sizes.
- The curves for the same CoV are identical in Figure C.6(a) and in Figure C.7(a) and also in Figure C.6(b) and Figure C.7(b). The overestimation is a function of the analysis method (5%ile by non-parametric methods or log-normal fit) and not the property being measured (5%ile strength or 5%ile MoE).



(a) non-parametric estimation of 5%ile



(b) estimation of 5%ile using log-normal



(c) estimation of 5%ile using mean of log-normal distribution and given CoV

Figure C.7 – Over-estimation of 5%ile strength using full data set

Figure C.7 shows that the overestimation of the 5%ile for sample sizes of 20 or greater is less than 1 or 2 % for the log-normal fit methods.

The over-estimation of a 5%ile where the full data set is used has a logical explanation. Particularly where small sample sizes are used, it is likely that the tails of the distribution will not be represented in every sample. The mean value of the sample will be close to the production mean, but the outer extremities of the sample will be closer to the mean than it is for the complete production. It is therefore likely that all percentiles (including the 5%ile) calculated from the sample will be closer to the mean than those of the production.

The implication of this finding is that where small samples are taken, the general trend is for an over-estimation – giving the appearance that the production properties are quite satisfactory. However when larger samples are taken (even by amalgamating all of the smaller data sets), the over-estimation is smaller, making it appear that the production did not have the same margin of safety that the test results from the succession of smaller samples indicated.

C.4.1.3 Estimation of 5%ile strength from tail data

In contrast to the systematic over-estimation of 5%ile strength by analyses that used the whole of the data, Figure C.8 shows that there is a systematic under-estimation of the 5%ile where the distribution is fitted only to the tail for small sample sizes, but that for larger sample sizes, the property is often over-estimated.

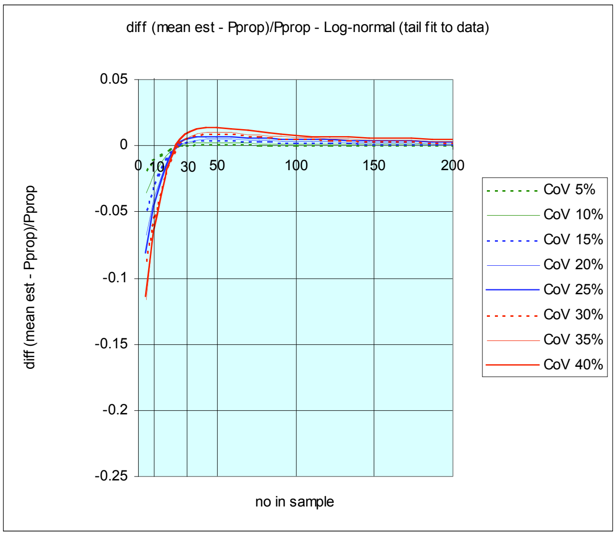
Figure C.8 shows that:

- For small sample sizes, the under-estimation is very significant at up to 10%. (The sample size is the number of tested pieces, whether or not they failed or were included in the tail.)
- Typically, for sample sizes of 50 or more, there is a small over-estimation (much less than 5%).

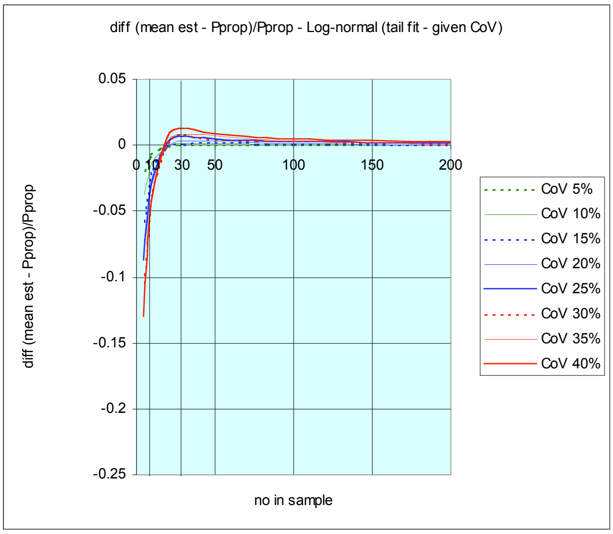
The under-estimation for small sized samples is because with a small total sample size, there is a small probability of having test values in the lower tail. In many cases, the 5%ile cannot be estimated at all from the tail, because there were not enough data points. However, in some simulations of small sample sizes, the sampling program had thrown up a number of very low points, and an evaluation of the 5%ile was possible.

In this way the data was biased – no value if the sampling gave generally higher strength pieces, and a value if the sampling gave generally lower strength pieces. This effect tended to disappear for sample sizes of 30 to 50. For samples sizes of 50 or more, the true character of the distribution could be seen:

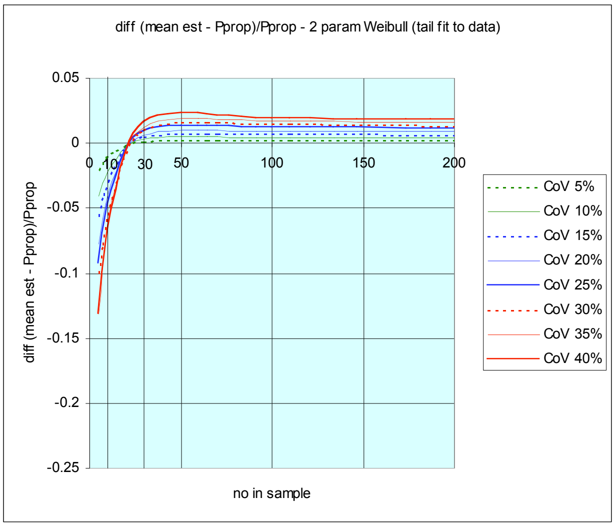
- Fitting data to log-normal distributions tended to give little or no over-estimation for very large samples sizes in a similar trend to the full data set discussed in Section C.4.1.2.
- Fitting data to 2 parameter Weibull distributions gave a consistent but small over-estimation due to the shape of the curve at the tail.



(a) estimation of 5%ile using log-normal tail fit



(b) estimation of 5%ile using log-normal tail fit with given CoV



(c) estimation of 5%ile using 2 parameter Weibull tail fit

Figure C.8 – Over-estimation of 5%ile strength using tail data

C.4.2 Confidence in curves fitted to the results of the simulations

Each of the sets of simulations returned a confidence limit for the estimation of the production property. For each analysis method and each confidence level, there were a number of evaluations for:

- Different values of CoV for the production, and
- Different numbers of specimens in the sample.

An equation of the form given in (*eqn C.24*) was fitted to the analysis results:

Confidence Limit on Production property (L) =

$$L = X\left(1 + A\frac{CoV}{\sqrt{n}}\right) \qquad (eqn\ C.24)$$

(*eqn C.24*) was fitted to the confidence limits from the different simulations that made up each analysis method and each confidence level. For strength evaluations, there were 56 combinations of different CoV and sample size, and for stiffness evaluations, there were 35 combinations.

The confidence limits from the simulations could be compared with the confidence limits predicted by the equation, and a correlation coefficient (r^2) found. In (*eqn C.24*) $\sqrt{n}$ can be written as $(n)^{0.5}$, with the power of n as 0.5. However, it was possible to see whether a different power of n gave a better fit to the data. For each set of results, the optimum power of n was found.

Table C.3 shows typical results for fitting the equation to the data.

Table C.3 – Curve fitting to simulation results

5%ile strength from non-parametric analysis					
Confidence level	0.95	0.9	0.85	0.8	0.75
Optimum exponent of *n*	0.484	0.506	0.526	0.545	0.5698
A	-3.69789	-3.07219	-2.65129	-2.30915	-2.02086
r^2	0.991	0.998	0.999	0.998	0.996
	-3.698	**-3.072**	**-2.651**	**-2.309**	**-2.021**
5%ile strength from log-normal analysis					
Confidence level	0.95	0.9	0.85	0.8	0.75
Optimum exponent of *n*	0.536	0.544	0.554	0.563	0.576
A	-2.65855	-2.10389	-1.73106	-1.43076	-1.17217
r^2	0.994	0.993	0.991	0.988	0.987
	-2.658	**-2.104**	**-1.731**	**-1.431**	**-1.172**
5%ile strength from log-normal analysis using given CoV (prodn value +/- 5%)					
Confidence level	0.95	0.9	0.85	0.8	0.75
Optimum exponent of *n*	0.395	0.374	0.353	0.333	0.310
A	-2.16558	-1.80641	-1.56428	-1.37204	-1.20416
r^2	0.994	0.990	0.9851	0.978	0.967
	-2.166	**-1.806**	**-1.564**	**-1.372**	**-1.204**
Average MoE from non-parametric analysis					
Confidence level	0.95	0.9	0.85	0.8	0.75
Optimum exponent of *n*	0.497	0.499	0.500	0.500	0.504
A	-1.64111	-1.28298	-1.03858	-0.84749	-0.6783
r^2	0.999	0.998	0.999	0.996	0.995
	-1.641	**-1.283**	**-1.038**	**-0.847**	**-0.678**

Table C.3 shows that in each case, the optimum exponent of n is close to 0.5, for all of the fits except the 5%ile strength from the log-normal distribution with a given CoV (within +/- 5% of the Production value). In the case of the analysis, which showed a different optimum exponent, the result was not particularly sensitive to the actual value, with the results for an exponent of 0.5 also giving a high correlation between the equation and the simulation results. An exponent of 0.5 was used for all relationships presented in this report.

The analysis was performed on each model considered, and the high correlation coefficient confirmed the form of the equation. Any analysis method that gave an r^2 for this correlation of less than 0.65 was not included in the outcomes of this report. The analysis methods presented in the outputs had an r^2 of more than 0.95.

C.4.3 Validation of the Outputs

There were a few outputs for which confirmation was available:

- Where a mean property is estimated, the confidence limits can be found by standard statistical methods[1,2,10,11,12]. It was possible to compare the values found using the simulations with the standard confidence limits.
- Where a 5%ile was sought, this was not possible, but there are a few cases where a confidence limit on 5%ile strength is given in Standards or other literature.

C.4.3.1 Validation of CL for mean values

The form of the equation for the Confidence Limit *(eqn C.24)*, is the same as for determination of confidence limits for the mean value from a normal distribution. The values of A from the estimation of the mean of the MoE, (where the values for A from the study were essentially the same for non-parametric, normal and log-normal distribution analyses) can be compared with the appropriate z values from the normal distribution.

Table C.4 – Comparison of CL for mean

Confidence level	0.95	0.9	0.85	0.8	0.75
Normal distribution (z)	-1.645	-1.282	-1.036	-0.842	-0.674
Study (A)	-1.649	-1.290	-1.045	-0.854	-0.686

Table C.4 shows that the methodology used in the simulations and subsequent analyses produced values for the confidence limit relationships that compared well with accepted statistical values for estimation of the mean.

In addition AS4063[10] presents an equation for finding the 75% confidence limit of average MoE as:

$$CL_{0.75} = \overline{E}\left(1 - 0.7\frac{CoV}{\sqrt{n}}\right)$$

In this case, the –0.7 is essentially the same value as the –0.69 in the study result. The Standard appears to use accepted statistical confidence calculations for MoE.

C.4.3.2 Validation of CL for 5%iles

There are no accepted statistical values for confidence limits on the estimation of the 5%ile from various distributions. However, there are two confidence estimates that have been accepted in timber testing, and they can be compared with the results of this study:

- US standards present the 75% CL on estimates of 5%ile strength of data obtained from tail fitting of a 2 parameter Weibull distribution as
  $$CL_{0.75} = x_{0.05}\left(1 - 2.7\frac{CoV}{\sqrt{n}}\right)$$ with the CoV as the tail CoV from the Weibull distribution and n as the total number of tests (not just the tail).

- European standards present the 75% CL on estimates of 5%ile from a log-normal analysis of the full data as

$$CL_{0.75} = x_{0.05}\left(1 - 1.25\frac{CoV}{\sqrt{n}}\right)$$ with the CoV from the log-normal distribution of the full data set.

A comparison can be made with the results of the simulations and analyses used in this study. This is shown in Table C.5.

In this case, there were no adjustments made for under-or over-estimation of the production properties. This differs from the values calculated for the remainder of the report in which the confidence limit was compared with the actual production value rather than the mean estimate. Table C.5 shows data that was compatible with the data believed to be the foundation of the overseas analyses.

Table C.5 – Comparison of CL for 5%ile strength

Confidence level	**0.75**	**0.75**
Origin	US[7]	Europe[17]
Analysis	2 parameter Weibull	Log-normal
CoV	Tail	full
N	Full	full
Adjustment for over-estimation	No	no
Value in Standards (*A*)	**~-2.7**	**-1.25**
Study (*A*)	**-2.281**	**-1.044**
Ratio (Standards/Study)	**1.18**	**1.19**

The standards seem to have a consistent factor of around 1.18 over the study results. Some conservatism in addressing the results of strength of timber has been built into the Standards.

The outcomes of this study did not incorporate any additional conservatism in the confidence limits presented for strength.

C.4.4 CoV for tail fitting 2 parameter Weibull distributions

It is well acknowledged that the CoV from a tail fit to a 2 parameter Weibull distribution gives a lower CoV than the CoV for the whole distribution[7]. Previous studies have indicated that it is about half of that for the whole distribution.

The study established a relationship between the CoV of the whole of the data and the appropriate CoV for the 2 parameter Weibull fit to the tail. This enabled comparisons of A values for (*eqn C.24*) between 2 parameter Weibull tail fits (where the CoV is less than for other methods).

The results of the simulations gave both the CoV for the whole of the data and the CoV determined from the 2 parameter Weibull fit through the tail of the data. A unique relationship was established that was independent of the CoV of the timber being sampled. The ratio varied with the proof stress (or the maximum stress used to define the tail of the distribution).

Figure C.9 shows the relationship between the two CoVs as a multiplier plotted against the probability of the proof load – *pr(proof)*. (For example, if the proof load is set at the 15%ile strength, then *pr(proof)* is 15%.) The data from the study is shown as points, and the thick black line is the line-of-best-fit through all of the test data.

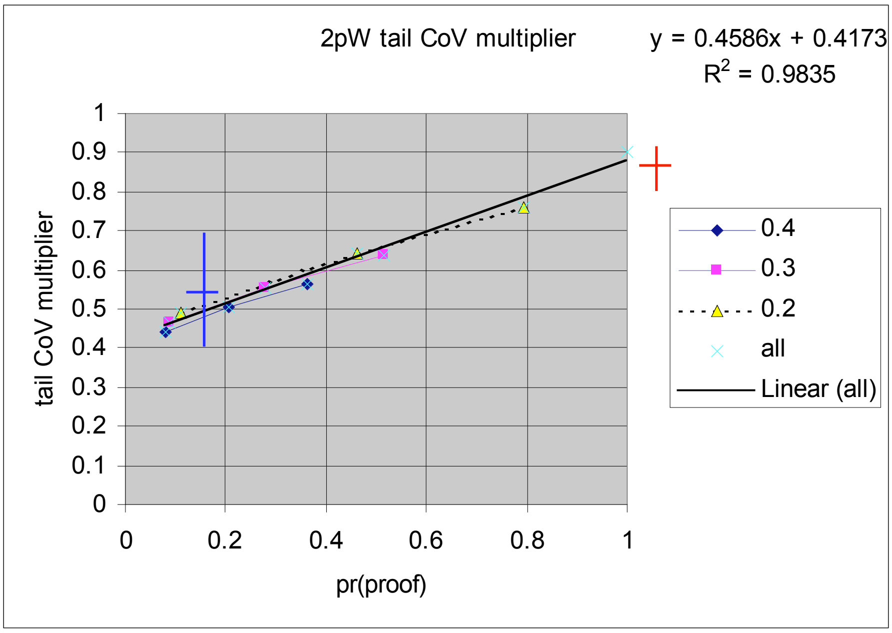


Figure C.9 Tail multiplier for 2 parameter Weibull tail CoV

The 2 parameter Weibull tail CoV is found by multiplying the CoV for the whole of the data by a multiplier given by (*eqn C.24*). This expression is a function of the probability of the proof stress in the whole of data distribution.

$$Mult_{CoV} = 0.4586 pr(proof) + 0.4173 \qquad (eqn\ C.25)$$

For this equation, the following key points result:

- Taking the tail as the lower 15% of the data, gives the value of the multiplier as 0.486, or very close to 0.5.
- Taking the whole of the data (or fitting to 100% of the data), gives the value of the multiplier as 0.876 or very close to 0.9.

The analysis of the 2003 data for all radiata pine mills MGP10 gave the following values:

- Multiplier for a tail to 15% is 0.58, with values for individual mills ranging from 0.39 to 0.7. This data is shown as a blue cross in Figure 13.
- Multiplier for 2 parameter Weibull fit to the entire data is 0.88, with values for individual mills ranging from 0.85 to 0.93. This data is shown as a red cross in Figure 13.

This finding can be used to estimate the CoV that should be used in fitting a 2 parameter Weibull distribution to the tail of some data, knowing that the CoV for the product normally has a given value.

C.4.5 Proof testing

A number of mills make use of proof testing as part of their monitoring programs. (This is different from the proof testing provisions of AS/NZS4490 – used in Periodic Monitoring.) In proof testing for daily testing, while each sampled piece is loaded, only the very weakest pieces are broken. When the test reaches a stress at which it is clear that the specimen will not have a strength in the tail of the distribution, the test is stopped. This stress is known as the proof stress, and it is generally just a little higher than the expected 5%ile of the timber. Testing in this way produces MoE data for each test specimen, but a reduced number of strength data. There is no strength data for pieces where the proof stress was attained without a failure and the test was stopped.

In these cases, it is only possible to fit a distribution through the tail of the data represented by the results of failures. (All of the strengths in the tail fit are less than the proof stress.)

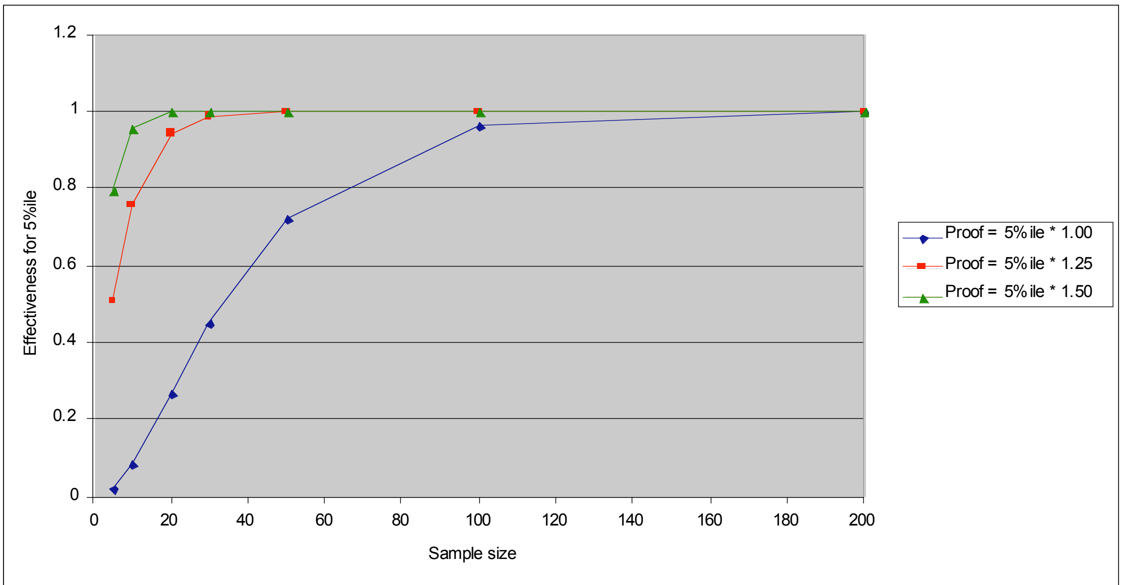


Figure C.10 – Effectiveness of determining 5%ile from tail fits with different proof loads
(log-normal fit averaged over different CoVs)

The estimate of the 5%ile strength of the test samples is affected by the level of proof stress used:

- Where only the tail is used to fit a distribution, there are not many data points to enable a good fit to the data. While the fit may match the character of the tail well, if the tail is affected by one or two samples, then the fit may not represent the rest of the distribution particularly well. The lower the proof stress, the fewer points there are in the data set.
- Where the proof stress is close to the 5%ile value, there may not be enough failure results from small test samples to enable calculation of the 5%ile value.

Figure C.10 shows a plot of the effectiveness of estimating the 5%ile strength from tail data using different proof stresses. The value plotted is the fraction of successful evaluations when the proof stress is the given ratio of the actual 5%ile of the data. It shows that:

- The proof stress needs to be at least 1.25 of the actual 5%ile to have reasonable success.
- For proof testing, a sample size of at least 20 is required to give at least a 90% effectiveness of evaluations.

Figure C.10 was presented for a log-normal fit to the tail of the distribution, and the results for the 2 parameter Weibull fit showed a similar trend, but with lower effectiveness for small sample sizes.

The effectiveness of the determination was also a function of the CoV of the material. The method was more effective where the CoV was smaller. This is illustrated in Figure C.11 which shows similar information for a proof load of 1.25 times the 5%ile but plotted for different CoV materials.

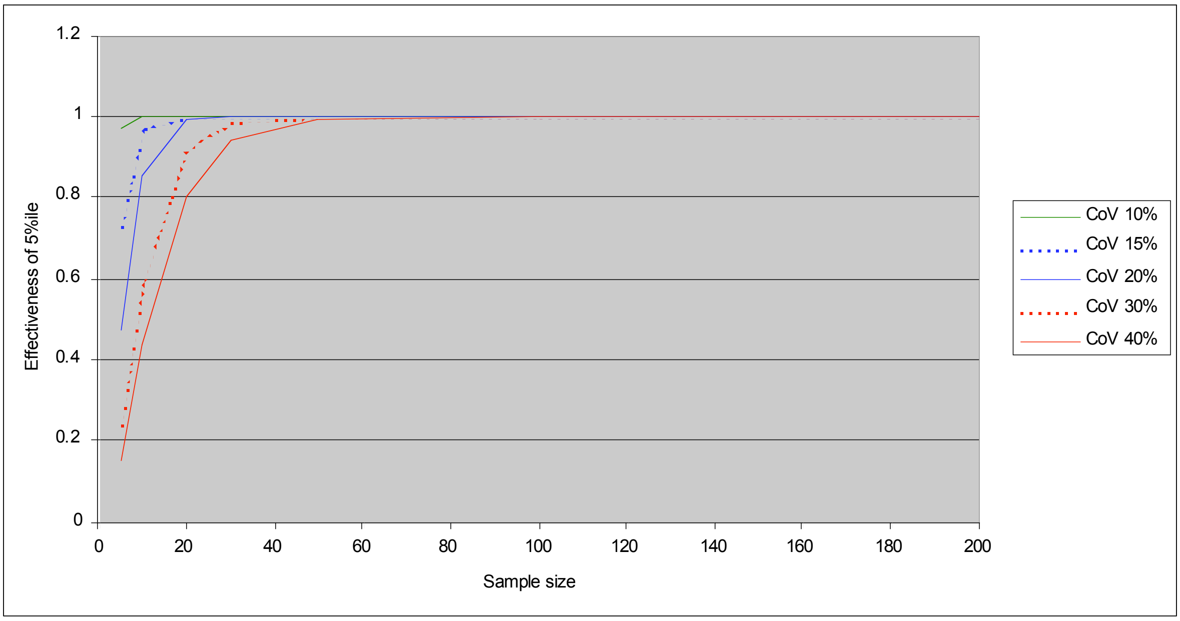


Figure C.11 – Effectiveness of determining 5%ile from tail fits on material with different CoVs
(log-normal fit for a proof load of 1.25 times 5%ile)

These plots confirm the fact that tail fitting may have very low reliability with small data sets. A minimum size is 50 tests, with 100 more reliable.

In the data presented for 2 parameter Weibull tail fitting, the 15%ile stress is chosen as the proof stress.

Table C.6 shows the variation in A (from *eqn C.24*) with different levels of proof load. It shows that the values decrease as the proof stress increases. This indicates that there is less sampling error in the results for higher proof stresses.

Table C.6 – error from larger proof stress (Indicated by *A* in *eqn C.24*)

Confidence level	0.95	0.9
Proof stress = 5%ile * 1.00	-4.06	-2.87
Proof stress = 5%ile * 1.25	-3.03	-2.40
Proof stress = 5%ile * 1.50	-2.98	-2.38

C.4.6 CUSUM simulation

Simulations of a single CUSUM process were performed in order to demonstrate the confidence producers could reasonably have in its ability to respond to problems in products.

CUSUM is a method for giving a binary (pass/fail) result for properties of any production process. There are international standards that detail it. However, as a binary process, it is not suited to the analysis methods presented in this project. It is used in the timber industry in Australia and elsewhere, so a simple comparison with the methods outlined in this project is given in this section.

CUSUM is one of a number of statistically based Quality Control tools. It is a QC method that has been widely used in the Northern hemisphere and by a number of Australian and New Zealand mills. It is used because of the following reasons:

- It has been codified for structural timber (overseas)[13]
- It uses relatively small sample numbers to give a binary result
- It requires very little statistical knowledge from managers of timber grading operations
- It is an off-the shelf QC method.

C.4.6.1 CUSUM for MoE

The simulation of CUSUM for MoE involved generation of five random test samples from data with a known average MoE and a given CoV. For each simulation, the data was subjected to the standard CUSUM for variables procedure[14]:

- The average MoE of the test specimens was found (E_{avg}).
- A SUM was evaluated from the previous CUSUM score, E_{avg} and the CUSUM variables.
- The CUSUM algorithm was used to evaluate the new CUSUM score.
- The CUSUM score enabled an interpretation as to whether or not the process was 'under control'.

The results of the analysis targeting MGP12 gave plots of the type shown in Figure C.12. Here the probability of the production being declared 'under control' is plotted against the average MoE of the population used to draw the samples. 95% probability means that there is a 95% chance of the production being declared 'under control'.

Figure C.12 shows that if the production MoE drops appreciably, the CUSUM system will reject it quite quickly.

- Where the production average MoE has fallen to a value less than 8.5 GPa (67% of the design value), then there is a probability of less than 96% that it will be classed as 'under control' by the CUSUM on the first shift after the change in properties. This massive reduction in properties has a 0.04% chance of passing on the second shift, so would almost certainly be caught on the second shift after the drop.
- The red line in Figure C.12 shows the performance of material in which the MoE has fallen to 11 GPa (87% of the design value). Here the first shift result has a 100% chance of being classes as 'under control', but after the second shift from the fall in properties, the CUSUM has 96% of being classed as

'under control'. By the third shift the probability is 66%, and after 10 shifts it is less than 0.1%.

- The blue line in Figure 16 is set at an average shift MoE of 12 MPa (95% of the design value). In this case, the probability of the first three shifts being classed as 'under control is more than 99%. It is only after 5 shifts that there is less than 95% confidence in having the production accepted.

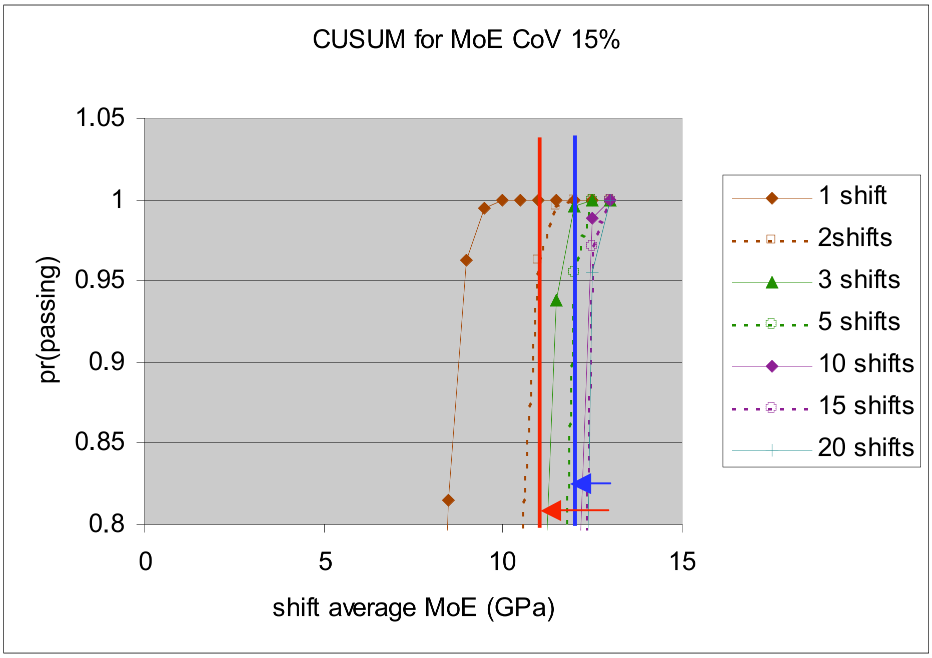


Figure C.12 – probabilities of MoE CUSUM accepting production

Table C.7 presents the 95% acceptance level for the production being classed by CUSUM as 'under control' for MGP12. This table gives the level that the average MoE for the shift has to drop to in order to have only 95% confidence of gaining an 'under control' outcome from the CUSUM after the designated number of shifts.

- For example for a CoV of 15%, after 2 shifts, the CUSUM result has a 95% chance of being 'under control' where the average MoE of the shift is 10.8 GPa or more.

Table C.7 – shift average MoE with 95% probability of being 'under control'

CoV	1 shift	2shifts	3 shifts	5 shifts	10 shifts	15 shifts	20 shifts
20	9.2	11.3	11.9	12.3	12.7	12.8	12.8
15	8.9	10.8	11.7	12	12.3	12.4	12.5
12	8.7	10.7	11.4	11.9	12.3	12.4	12.4
10	8.6	10.7	11.3	11.9	12.3	12.4	12.4
8	8.4	10.7	11.3	11.8	12.1	12.2	12.3

Table C.7 shows that as long as the MoE is above the design value on a sustained basis, there is little chance of a rejection of product. On the other hand, shifts that have dropped to less than 10 MPa do not have 95% confidence in being passed after the second consecutive shift at that level.

C.4.6.2 CUSUM for strength

The simulation of CUSUM for strength involved generation of five random test samples from data with a known 5%ile strength and a given CoV. For each simulation, the data was subjected to the standard CUSUM for attributes procedure[14]:

- The number of failures at a strength less than the threshold strength were counted (*N*).
- A SUM was evaluated from the previous CUSUM score, *N* and the CUSUM variables.
- The CUSUM algorithm was used to evaluate the new CUSUM score. The algorithm differed slightly depending on whether the previous CUSUM was 'under control' or not.
- The CUSUM score enabled an interpretation as to whether or not the process was 'under control'.

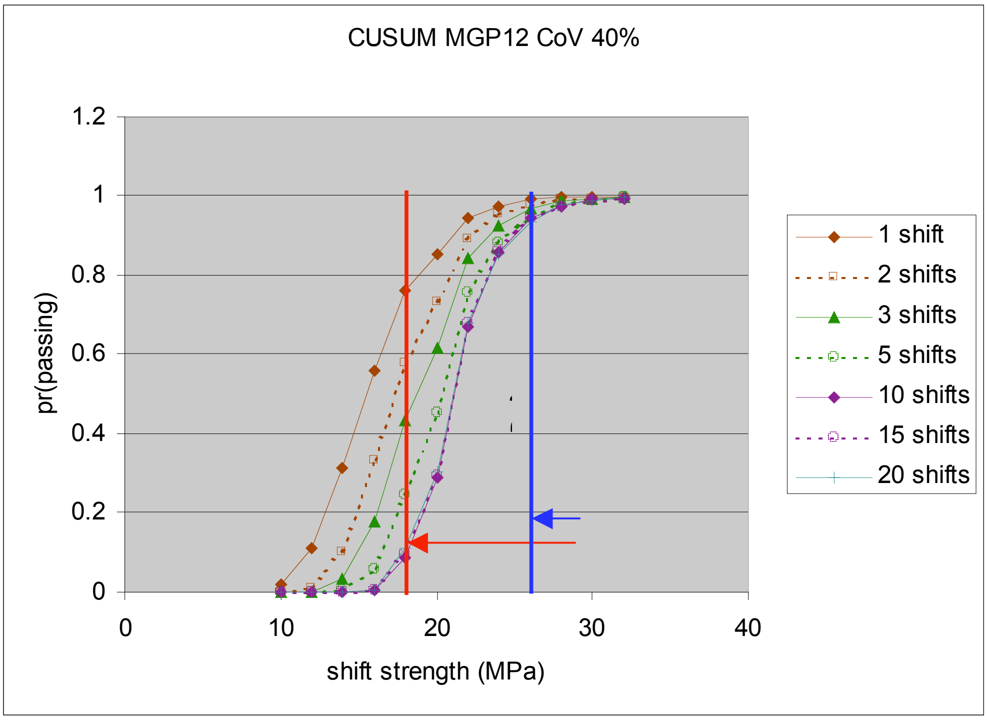


Figure C.13 – probabilities of strength CUSUM accepting production

The results of the analysis targeting MGP12 properties gave plots of the type shown in Figure C.13. Here the probability of the production being declared 'under control' is plotted against the 5%ile strength of the population used to draw the samples. 95% probability means that there is a 95% chance of the production being declared 'under control'.

Figure C.13 shows that if the shift strength is very low, CUSUM will reject it quite quickly, but if the shift strength is quite close to the design value, it takes much longer to be rejected.

- A shift 5%ile strength to 12 MPa has only an 11% chance of being classed as 'under control' in its first shift. (This is equivalent to saying that it has an 89% chance of being rejected.
- The red line in Figure C.13 illustrates the scenario of the product 5%ile strength dropping suddenly to 18 MPa (64% of the design value). Under these

circumstances, there is a 76% chance of passing the first shift. However, the probability of passing the second shift after the drop is less (58%) and lower again after three shifts (44%). The probability of it still being classed as 'under control' by CUSUM after 20 shifts is 11%.

- The blue line in Figure C.13 illustrates the scenario of the product 5%ile strength being maintained at 24 MPa (86% of the design value). In this case, there is a 97.5% chance of the production being classed as 'under control' in the first shift at the new value. (It only has s 2.5% chance of being caught.) The probability of passing the production decreases slightly with each continuing shift, but after 10 shifts, it has stabilised at 85%. It has a relatively low probability of rejection.

As the shift strength gets closer to the design strength (in this case, 28 MPa) it is much more likely to be accepted by the CUSUM criteria[15]. Table C.8 presents the shift strength for production that has a 95% probability of being accepted.

Table C.8 – shift strength with 95% probability of being 'under control'

CoV	1 shift	2 shifts	3 shifts	5 shifts	10 shifts	15 shifts	20 shifts
40	22	24	25	26	27	27	27
35	23	24	25	26	27	27	27
30	23	24	25	25	26	26	26
25	23	24	25	25	25	26	26

Table C.8 shows that there is a high probability of acceptance of material for a number of shifts where the strength is a little less than the design strength, but not appreciably lower than the design value.

Table C.8 does not appear to be very sensitive to the CoV of the production material. At a probability level of 50%, there is a little more sensitivity, but the process does not vary much over the range of CoVs expected in sawn timber.

C.4.6.3 Comparison between CUSUM for strength and MoE

The red line in Figure C.12 (87% of the design MoE) can be compared with the blue line in Figure C.13 (86% of the design strength).

- For the MoE (the red line in Figure C.12) the first shift result has a 100% chance of being classed as 'under control', but after the third shift the probability is 66%, and after 10 shifts it is less than 0.1%.
- For the strength (the blue line in Figure C.13) the first shift result has a 97.5% chance of being classed as 'under control', but after 10 shifts, it has stabilised at 85%. It has a relatively low probability of rejection.

A fall in MoE value has a much higher chance of being caught compared with a fall of the same relative magnitude in strength. In both cases, there is a small chance of detection in the first shift unless the fall in properties is of the order of one grade.

The outputs from CUSUM from the same examples can also be compared with outputs from other methods of analysing the same data. For MoE:

- For the case of an evaluation at the end of a single shift – using only five data points.

  - Using CUSUM, one shift at 11 GPa (87% of the design MoE) gave a 100% chance of being classed as good, or a 0% chance of being detected as a problem.
  - Had the non-parametric average MoE been found from the same 5 samples, then the result would have had a 1% chance of being greater than 12.7 GPa – clearly indicating a problem. Another way of considering it, is that for a 90% Confidence Limit, the Test Comparison Value would be 13.91 GPa and the student t test gives a 0.8% probability of a sample of 5 drawn from a production with a mean of 11 GPa and a CoV of 15% exceeding that value.
- For the case of an evaluation at the end of two production shifts after which a total of 10 data points are available and a decision will be made on the acceptance of both shifts taken as a group.
  - Using CUSUM, two shifts at 11 GPa (87% of the design MoE) gave a 96% chance of being classed as good, (based on a total of 10 pieces tested) or a 4% chance of being detected as a problem.
  - Had the non-parametric average MoE been found from the same 10 samples, then the result would have had a 0.1% chance of being greater than 12.7 GPa – clearly indicating a problem. Another way of considering it, is that for a 90% Confidence Limit, the Test Comparison Value would be 13.53 GPa and the student t test gives a 0.05% probability of a sample of 10 drawn from a production with a mean of 11 GPa and a CoV of 15% exceeding that value.

Therefore, using the same data to find the non-parametric average gives better certainty in detecting problem batches compared with the CUSUM method. (In this case 95% confidence for the raw data compared with 4% after two shifts using CUSUM.)

A similar comparison can be made between the strength from CUSUM and the strength from a log-normal fit through the same data.

- Based on the results of a single shift:
  - Using CUSUM, a single shift at 24 MPa gave a 97.5% chance of returning an ‘under control’ result. This equates to a 2.5% chance of detecting a problem at the end of that shift in which 5 samples were tested.
  - Had a log-normal fit to the five points been used to estimate a 5%ile with a CoV of 35%, then there is a 27.1% chance of the 5%ile being greater than the 28 MPa design value, based on a population with a 5%ile strength of 24 MPa. The probability of exceeding a Test Comparison Value (for any Confidence Level) will be much less than this. There is clearly a problem indicated after one shift.
- Based on the results of 10 shifts. (Assuming that the analysis is performed after 10 shifts and a decision made on the quality of that 10 shifts as a block)
  - Using CUSUM, ten shifts at 24 MPa gave an 85% chance of returning an ‘under control’ result based on a total of 50 test specimens. This equates to a 15% chance of detecting a problem.
  - Had a log-normal fit to the fifty points been used to estimate a 5%ile with a CoV of 35%, then there is a 1.6% chance of the 5%ile being greater than the 28 MPa design value, based on a population with a

5%ile strength of 24 MPa. The probability of exceeding a Test Comparison Value (for any Confidence Level) will be much less than this. It is highly unlikely that the problem would remain undetected after 10 shifts using this method of analysis.

For both strength and MoE, the analysis of the data to estimate properties directly gives a more reliable outcome than the use of CUSUM on the same data in comparing both the results of a single shift and the results of a number of shifts taken together.

C.4.7 Finding mean MoE from MSG data

There is a high correlation between mean MoE determined from random position testing on random samples and the mean of the MSG averaged over the length of the same pieces, provided a sufficient number of pieces is used.

Figure C.14 shows a scatter plot of random position MoE and MSG averaged over the length of the same pieces. The data has been sourced from the 2003 MGP study for only 90×35 timber and covers a range of grades. It also shows the equality line. The data appears to be quite close to the equality line, though there is a scatter around the line. However, different relationships may exist for material from different resources.

It was possible to determine the ratio (mean MoE/avg MSG) for different groupings of the data. A very strong correlation was found between the character of the timber and the ratio. Figure C.15 shows a plot taken from the same data shown in Figure C.14 in which the ratio is plotted against (min MSG/avg MSG) for the same grouping of timber.

- Figure C.15 shows red squares and lines for the ratio (mean MoE / average MSG). The full red boxes are from the grouped data shown in Figure C.14, and the data plots in a straight line. As well, each of the mills in the 2003 study were used to derive points for their MGP10 grades and these are shown as open boxes. The linear relationship (*eqn C.26*) is shown as the red line.
- Figure C.15 shows blue diamonds for the calculated value of CoV for the grouped data that gave the red boxes in the same plot. The CoV of the ratio also is very linear. Again the CoV was evaluated for the ratio for MGP10 from each of the mills in the 2003 study, and plotted as open blue diamonds. Again, the mill data falls very close to the linear approximation. The linear relationship (*eqn C.27*) is shown as the blue line.

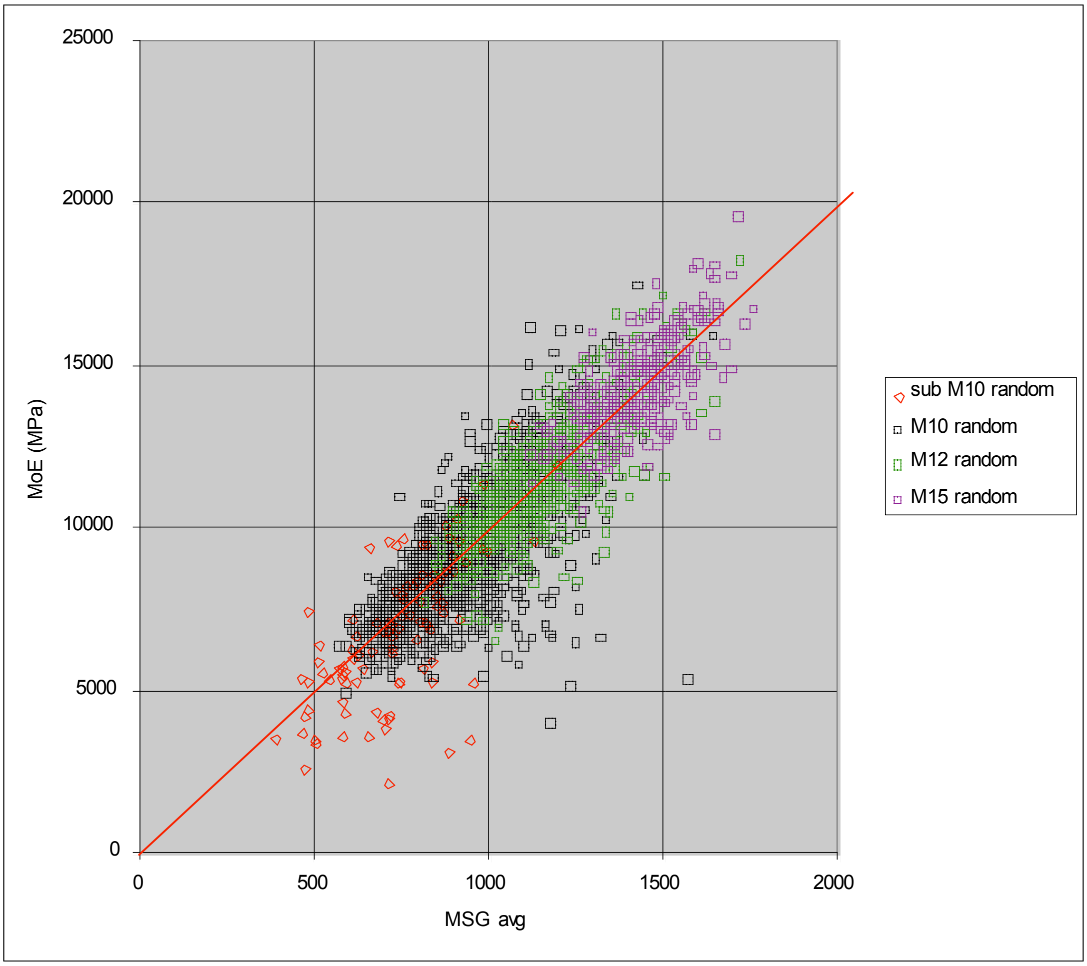


Figure C.14 – MoE vs MSG averaged over length

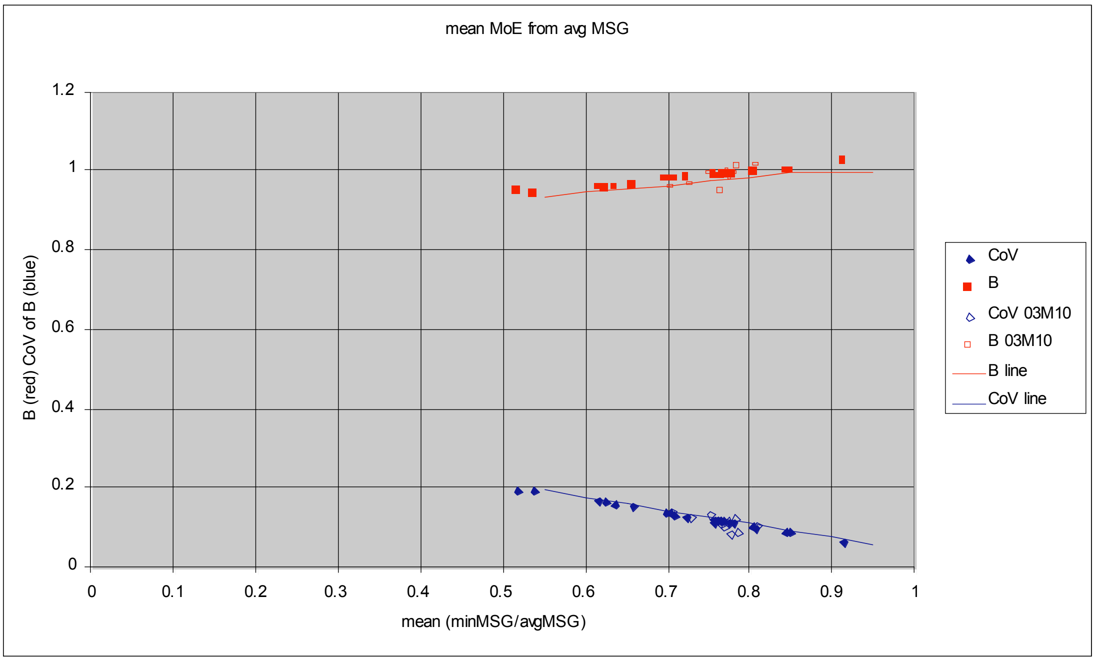


Figure C.15 – mean MoE prediction parameters from MSG information

The linear relationships plotted in Figure C.15 are given as (*eqn C.26*) and (*eqn C.27*)

$$\left(\frac{meanMoE}{avgMSG}\right) = B = 0.827 + 0.197\left(\frac{\min MSG}{avgMSG}\right) \qquad (eqn\ C.26)$$

$$CoV_{\left(\frac{meanMoE}{avgMSG}\right)} = 0.377 - 0.334\left(\frac{\min MSG}{avgMSG}\right) \qquad (eqn\ C.27)$$

The prediction of mean MoE is therefore:

$meanMoE = B\ avgMSG$ with B given in (*eqn C.26*)

The shift mean MoE can be estimated from the average MSG for the shift multiplied by *B* given by (*eqn C.26*). *B* is a function of the character of the product and it can be found from the average (min MSG) for the grade and the average (avg MSG) for the grade.

In order to estimate the confidence of the data, the CoV of the ratio is needed. It can also be found from the average (min *MSG*) for the grade and the average (*avg MSG*) for the grade. This gives rise to a slightly different form of the equation for the confidence limit as shown in (*eqn C.28*). When comparing it with (*eqn C.24*) it can be seen that it has the factor *B* involved.

$$\text{CL on Mean MoE} = CL_E = \overline{MSG}\ B\left(1 + A\frac{CoV}{\sqrt{n}}\right) \qquad (eqn\ C.28)$$

Here the CoV is the CoV of *B* and is given by (*eqn C.26*) and *n* is the number of pieces of wood that contribute to the average.